

QUANTIFYING THE ECONOMIC VALUE OF WEATHER FORECASTS: REVIEW OF METHODS AND RESULTS

Rick Katz

**Institute for Study of Society and Environment
National Center for Atmospheric Research
Boulder, CO USA**

Email: `rwk@ucar.edu`

Web site: `www.isse.ucar.edu/HP_rick.html`

Lecture: `www.isse.ucar.edu/HP_rick/pdf/ukkatz.pdf`

Quote

“You don’t get points for predicting rain. You get points for building arks.”

Lou Gerstner
(Former IBM CEO)

Outline

- (1) Motivation**
- (2) Prescriptive Decision-Making Framework**
- (3) Economic Value of Forecasts**
- (4) Prototypical Decision-Making Models**
- (5) Quality-Value Relationships**
- (6) Valuation Puzzles**
- (7) Review of Case Studies**
- (8) Discussion**

(1) Motivation

- Finley's Tornado Forecasts (1884)

Forecast	<u>Observed</u>	
	Tornado	No Tornado
Tornado	$n_{11} = 28$	$n_{10} = 72$
No Tornado	$n_{01} = 23$	$n_{00} = 2680$

-- 96.6% correct

(1) Motivation

- Finley's Tornado Forecasts (1884)

Forecast	<u>Observed</u>	
	Tornado	No Tornado
Tornado	$n_{11} = 28$	$n_{10} = 72$
No Tornado	$n_{01} = 23$	$n_{00} = 2680$

-- 96.6% correct (But 98.2% correct if never forecast tornado!)

- **Peirce's Skill Score** (“The Numerical Measure of the Success of Predictions”, *Science*, 1884)
 - Proposed “science of the method” measure for forecast evaluation
 - Model as if produced by two idealized observers
 - (i) Science of the method (Infallible observer)
 - (ii) Random guessing (Ignorant observer)

$$\text{PSS} = n_{11} / (n_{11} + n_{01}) - n_{10} / (n_{10} + n_{00}) \approx 0.549 - 0.026 \approx 0.523$$

Estimates “Hit Rate” minus “False Alarm Rate” ($-1 \leq \text{PSS} \leq 1$)

PSS = 0 if never forecast tornado

- “Utility” of Predictions

- Peirce proposed measure of “utility of method of prediction”

S = saving from predicting tornado (benefit of preparation, etc.)

l = loss from “unfulfilled” prediction (making preparation, etc.)

- Average profit per prediction:

$$S (n_{11} / n) - l (n_{10} / n)$$

where $n = n_{11} + n_{01} + n_{10} + n_{00} =$ total number of forecasts

- **Note:** Assumes make preparations if and only if given forecast of tornado

(2) Prescriptive Decision-Making Framework

- Elements of Decision Making

- Events ($\Theta = \theta$)

Adverse weather event (such as “Rain”)

- Actions (a)

Protection (to prevent or reduce impact of event)

- Consequences [$w(a, \theta)$]

Action-Event pair

- **Forecast Information System**

- **Forecast Weather ($Z = z$)**

Conditional probability distribution for event

$Z = z$ indicates forecast for particular occasion

Specifies conditional distribution of weather ($\Theta \mid Z = z$)

Systems viewpoint: Also need unconditional distribution of Z

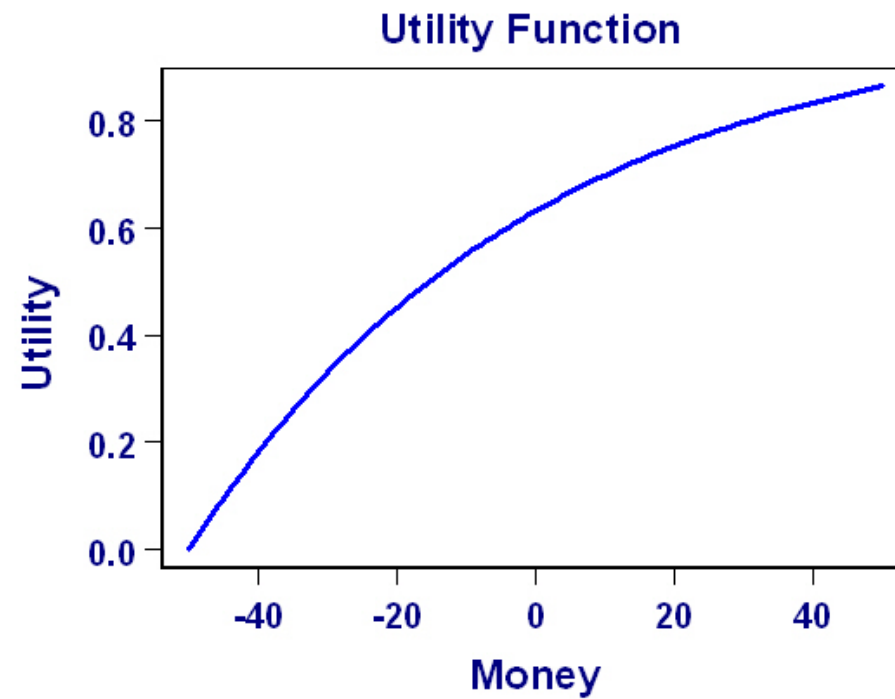
- **Climatological Information (“Climatology”)**

Unconditional probability distribution of Θ

(3) Economic Value of Forecasts

- Optimal Use of Forecasts

-- Utility function u



-- *Risk neutral*

u linear function (suffices to work in monetary terms)

-- *Risk aversion*

u concave function (figure above)

-- *Risk taking*

u convex function

-- Payoff (utility function evaluated at consequence)

$u[w(a, \theta)]$

-- Criterion for optimality

Maximize expected utility

Optimization procedure:

(i) For each possible action a , determine expected utility (weighted average with respect to conditional probability distribution for future weather as specified by forecast, $Z = z$)

$$E_{\Theta|Z} \{u[w(a, \Theta)] \mid Z = z\}$$

(ii) Select action a for this expected utility is a maximum

- **Economic Value of Forecasts**

- **Standard of Comparison**

- e. g., climatological information

- Other examples (“persistence”, alternative forecasting system)

- **Concept of Value of Imperfect Information (VOI)**

- Measure of how much better off decision maker is
(with vs. without forecasting system)

- e. g., increase in expected return (or reduction in expected expense)
for forecasting system as compared to climatology

-- General Definition of **VOI** (“Demand Value”)

Maximum amount, measured in same units as those in which consequence of decision measured, that decision maker would be willing to exchange for information system

Formal Definition of Demand Value:

$$E_Z(\max_a E_{\Theta|Z}\{u[w(a, \Theta) - \text{VOI}] \mid Z\}) = \max_a E_{\Theta}\{u[w(a, \Theta)]\}$$

Consistent with concept of contingent valuation in economics (“willingness-to-pay”)

(4) Prototypical Decision-Making Models

- **Background**

- Long tradition of use in valuing weather forecasts
- Still quite popular in valuing ensemble prediction systems
- Limited, if any, descriptive content

- **Cost-Loss Decision-Making Model**

- Static decision-making structure
- Only two possible actions
- Only two possible states of weather
- Assumes linear utility function

-- Expense Matrix [$w(a, \theta)$]

Action	<u>Weather Event</u>	
	Adverse ($\Theta = 1$)	Not Adverse ($\Theta = 0$)
Protect ($a = 1$)	C	C
Do Not Protect ($a = 0$)	L	0

If take protective action, then incur cost C

If do *not* protect and adverse weather event occurs, then incur loss L

- **Optimal Use of Forecasts**

- **Decision-Making Criterion**

Minimize expected expense

(i. e., assume linear utility function)

- **Optimal Policy**

Forecast probability of adverse weather event p

Expected Expense (*Protect*): C

Expected Expense (*Do Not Protect*): $(1 - p) 0 + p L = p L$

So protect if $p > C / L$

-- Finley's Tornado Forecasts

Estimated climatological probability:

$$\Pr\{\Theta = 1\} \approx 0.018$$

Estimated conditional probability distribution of Θ given Z :

$$\Pr\{\Theta = 1 \mid Z = 1\} \approx 0.280$$

$$\Pr\{\Theta = 1 \mid Z = 0\} \approx 0.009$$

Cost-loss ratio C/L presumably relatively small

(e. g., decision of whether to issue tornado warning for region)

- Economic Value of Forecasts

- Climatology

Climatological probability of adverse weather $p_A = \Pr\{\Theta = 1\}$

Minimal Expected Expense E_{CLIM}

$$E_{\text{CLIM}} = p_A L, \text{ if } p_A \leq C / L$$

$$E_{\text{CLIM}} = C, \text{ if } p_A > C / L$$

- Forecasts (with Minimal Expected Expense E_{FORE})

$\Pr\{\Theta = 1 \mid Z = z\}$ varies with $Z = z$ (i. e., dist. of conditional probs.)

$$\text{VOI} = E_{\text{CLIM}} - E_{\text{FORE}}$$

- **Other Prototypical Decision-Making Models**

- **Dynamic (instead of static)**

- e. g., assume loss L can be incurred at most once

- Criterion of minimizing total expected expense over some time horizon (perhaps discounted)**

- Optimal policy no longer protect if $p > C / L$ (change in threshold)**

- Consequent effect on forecast value as well**

(5) Quality-Value Relationships

- **Statistical Model for Generating Probability Forecasts**
(e. g., from perfect numerical weather prediction system)
- **Beta Distribution (for probability forecast p)**

Natural distribution for probabilities, $0 < p < 1$

Parameters r, s ($0 < r < \infty, 0 < s < \infty$)

Mean: $r / (r + s)$

Assume perfectly reliable probability forecasts

In particular, $p_A = r / (r + s)$

-- Climatology (i. e., no skill)

$$r \rightarrow \infty, s \rightarrow \infty \quad (\text{holding } p_A \text{ constant})$$

-- Perfect information

$$r \rightarrow 0, s \rightarrow 0 \quad (\text{holding } p_A \text{ constant})$$

-- Brier score (**BS**) $BS = E[(\Theta - p)^2]$

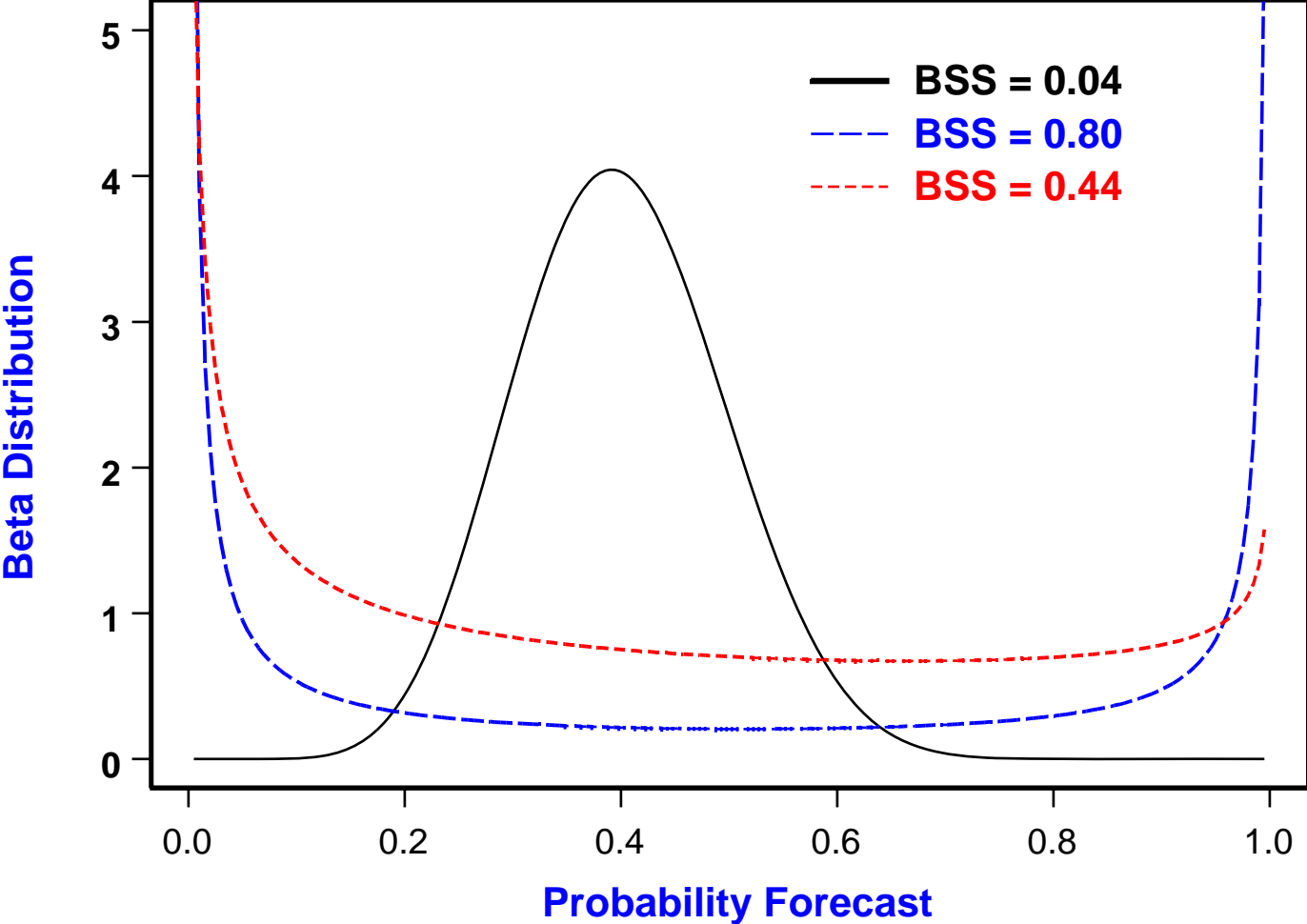
-- Brier skill score (**BSS**) $BSS = 1 - [BS / \text{Var}(\Theta)], \quad 0 \leq BSS \leq 1$

Climatology: $BSS = 0$

Perfect information: $BSS = 1$

Model based on beta distribution: $BSS = 1 / (r + s + 1)$

Forecast Systems: $p_A = 0.4$



- **Cost-Loss Decision-Making Model**

- Fix climatological prob. of adverse weather p_A (i. e., fix either r or s)

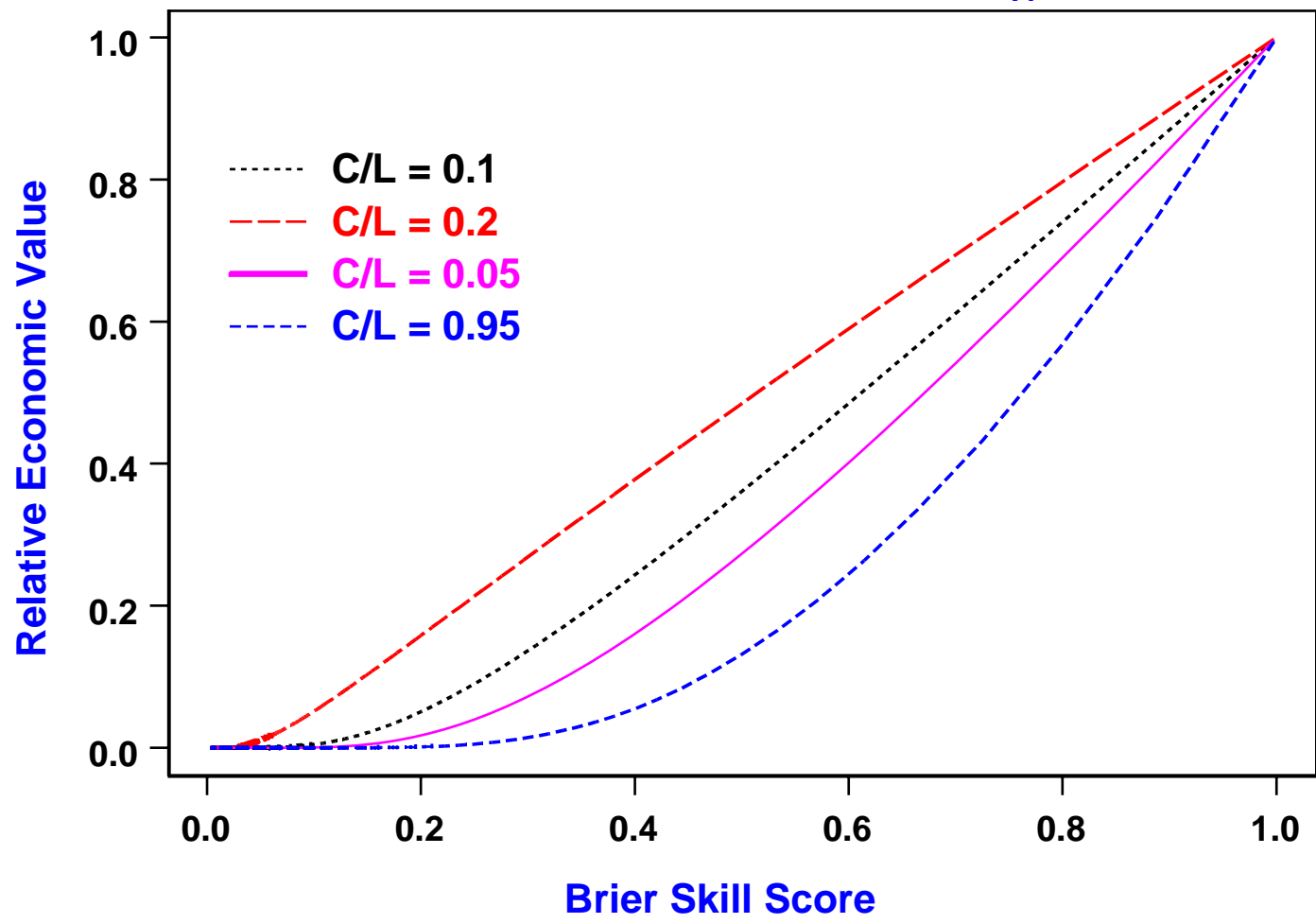
Fix cost-loss ratio C / L

- Vary skill of forecasting system (through parameter r or s of beta distribution) from no skill (i. e., climatology) to perfect information

- Examine how economic value of forecasting system changes (on relative scale from zero for climatology to one for perfect)

- Use expressions in Katz & Ehrendorfer (2006)

Quality-Value Relationships: $\rho_A = 0.4$



- **General Result**

- **Concept of “sufficiency”**

Criterion for comparison of “quality” of two forecasting systems (say issuing forecasts Z_1 & Z_2):

Z_2 is “sufficient” (i. e., at least as valuable as) for Z_1 if there exists a stochastic transformation T such that

$$f_1(z_1 | \theta) = \int T(z_1, z_2) f_2(z_2 | \theta) dz_2$$

where $f_i(z_i | \theta)$ denotes conditional density of forecast given weather for system i , $i = 1, 2$

(*Interpretation:* Additional layer of “randomness”)

- Theorem (David Blackwell, “Equivalent Comparisons of Experiments”, *Annals of Math. Stat.*, 1953)**

Quality measure consistent with sufficiency

Quality-value relationships must be non-decreasing

- Partial ordering**

(i. e., one forecasting system may be more valuable for some users, another one for other users)

(6) Valuation Puzzles

- **Quality-Value Reversals**

- **Ensemble prediction system**

Assume perfect numerical model

(apply statistical model for probability forecasts based on beta distribution)

- **But now only finite number m of ensembles available**

Generate these ensembles using underlying forecast probability p from beta distribution

Usual approach:

Take ensembles at “face value”

Suppose event occurs for k out of m ensembles

Estimate forecast probability as

$$\hat{p} = k/m$$

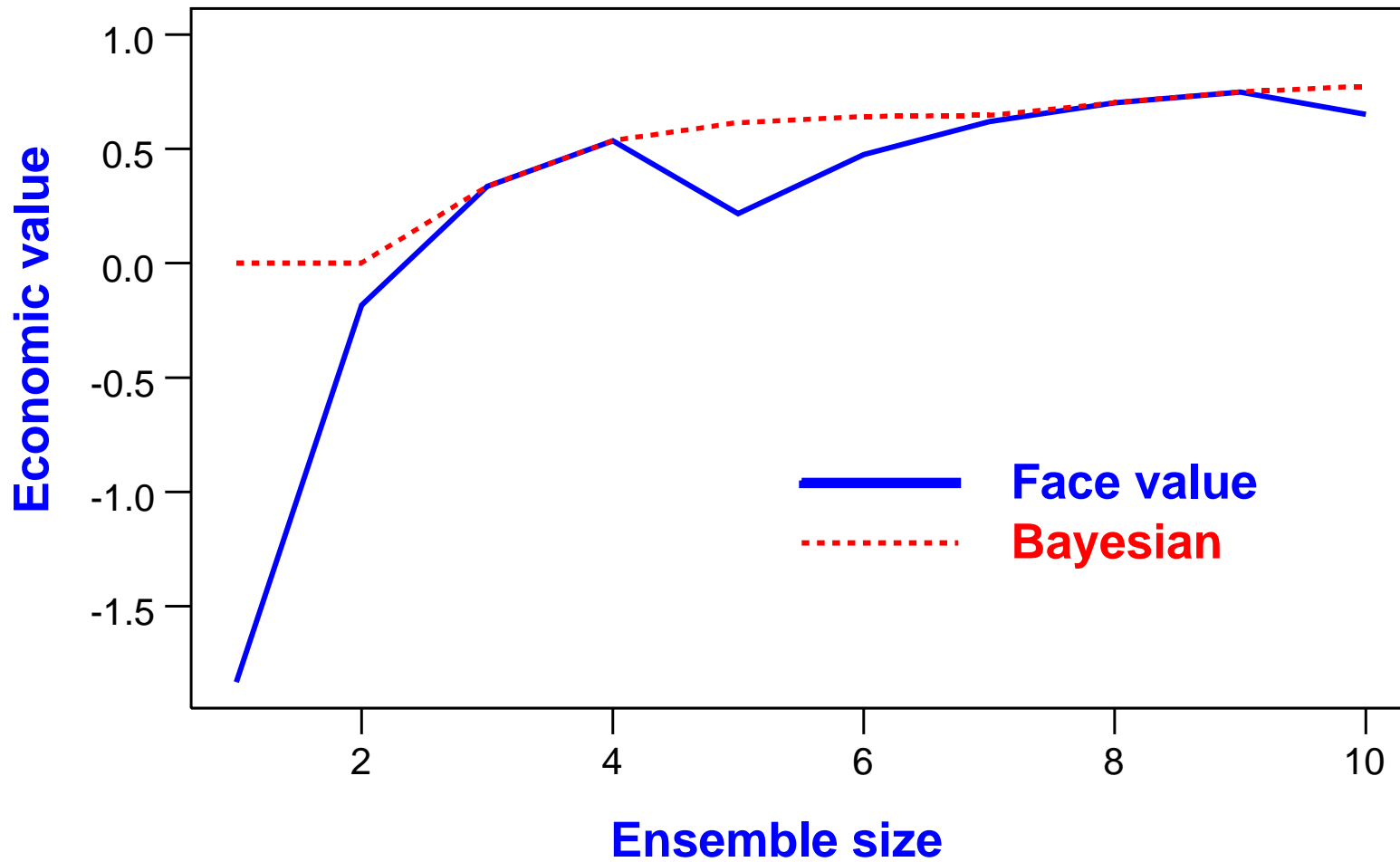
Alternative approach:

Bayesian (in which uncertainty in estimating p also taken in account)

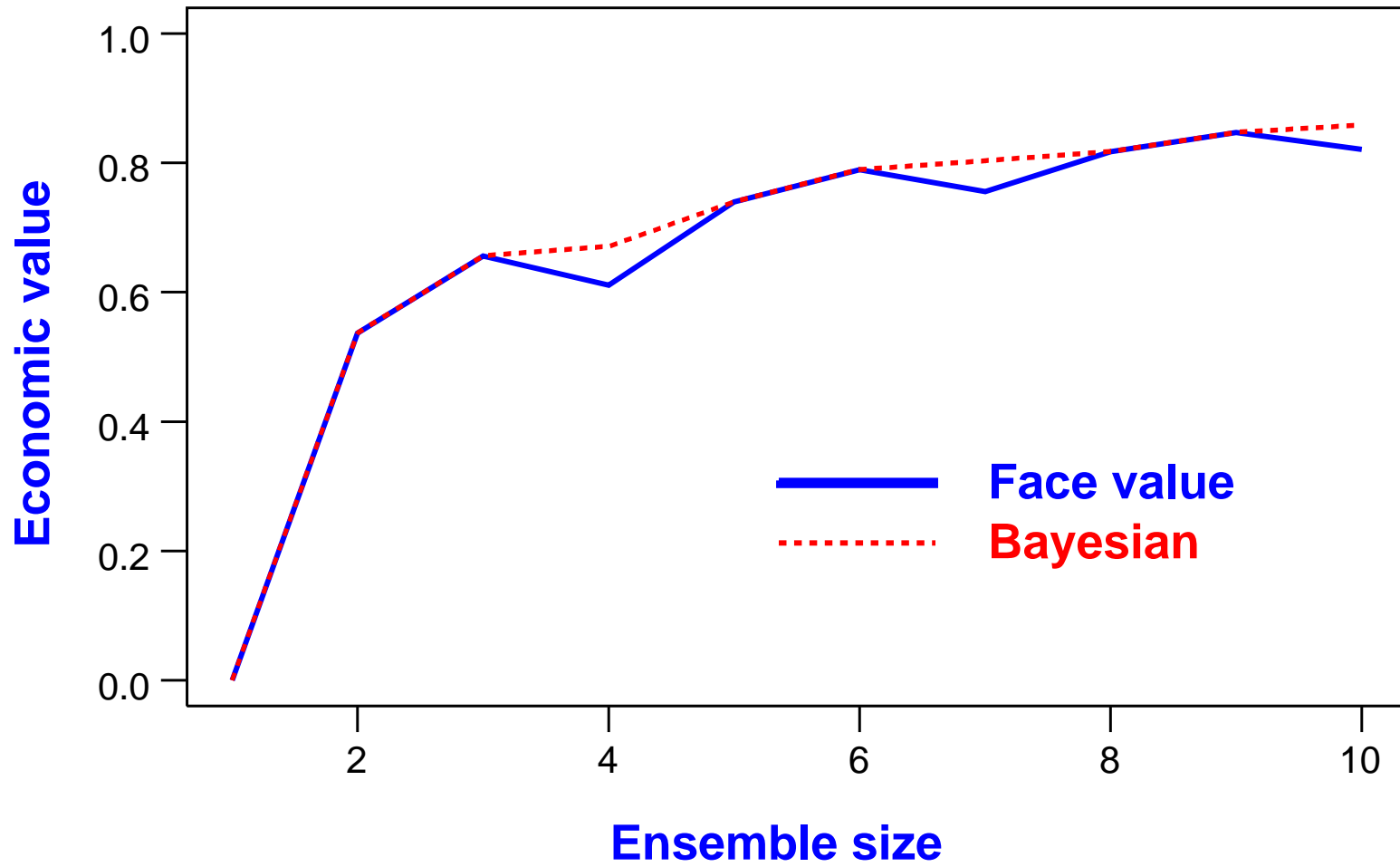
-- Cost-Loss Decision-Making Model

Usual prototypical model for valuing ensemble forecasts

$r = 0.75, s = 0.75, C/L = 0.2$



$r = 0.75, s = 0.75, C/L = 0.3$



- **Theoretical Results**

- **Attributes of decision setting & decision maker**

- Flexibility**

- Risk aversion**

- Wealth**

- Prior uncertainty**

- **No monotonic relationship between information value and any of these attributes**

(7) Review of Case Studies

- **Recent Case Studies of Economic Value**

-- www.isse.ucar.edu/HP_rick/esig.html

Any published prescriptive study in which economic value of weather or climate forecasts quantified

Earlier studies reviewed in chapter by Dan Wilks

(*Economic Value of Weather and Climate Forecasts*, edited by Katz & Murphy, Cambridge Univ. Press)

- Tables listing case study attributes**

 - Structure of decision problem**

 - Forecast characteristics**

 - Information Valuation**

- Sectors**

 - Agriculture (by far most studies)**

 - Energy**

 - Fishery**

 - Transportation**

- Forecast variables**

 - El Niño-Southern Oscillation (ENSO) phenomenon (most attention)**

-- Example table

<p>Study</p>	<p>Solow, A.R., Adams, R.F., Bryant, K.J., Legler, D.M., O'Brien, J.J., McCarl, B.A., Nayda, W., and Weiher, R. (1998). The value of improved ENSO prediction to U.S. agriculture. <i>Climatic Change</i>, 39, 47-60.</p>		
	<p>Structure of Decision Problem</p>	<p>Forecast Characteristics</p>	<p>Information Valuation</p>
	<p><u>Decision:</u> allocations among various crops <u>Dynamics:</u> no</p>	<p><u>Time Scale:</u> annual <u>Predictand:</u> El Nino <u>Format:</u> categorical <u>Type:</u> realistic, derived <u>Quality Changes:</u> yes</p>	<p><u>Baselines:</u> climatological and persistence <u>VOI, imperfect:</u> \$240-\$266 million/year (1995 US \$) <u>VOI, perfect:</u> \$323 million/year (1995 US \$) <u>Risk Treatment:</u> expected value</p>

(8) Discussion

- **Potential Value of Weather Forecasts**

- **Fairly well established**

Wide variety of situations

- **Realized Value of Weather Forecasts**

- **Very limited study**

Comparison with contingent valuation results (problematic)

Descriptive case studies

(rarely result in quantitative estimates of forecast value)