ECONOMIC VALUE OF FORECASTS

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Lecture: www.isse.ucar.edu/HP_rick/npskatz.pdf

QUOTES ON DECISION MAKING / INFORMATION

- "*You don't get points for predicting rain. You get points for building arks.*" **(Lou Gerstner)**

- "*Information is information, not matter or energy*" **(Norbert Wiener)**

OUTLINE

(1) Decision-Making Framework

(2) Economic Value of Forecasts

(3) Cost-Loss Decision-Making Model

(4) Quality-Value Relationships

(5) Valuation Puzzles

(6) Resources

(1) DECISION-MAKING FRAMEWORK

(1.1) ELEMENTS OF DECISION MAKING

- **Events ($\Theta = \theta$)**

-- Adverse weather event (such as "Rain")

- **Actions (*a*)**

-- Protection (to prevent or reduce impact of event)

- **Consequences [$w(a, \theta)$]**

-- Action-Event pair

(1.2) FORECAST INFORMATION SYSTEM

- **Forecast**

-- Conditional probability distribution for event

$Z = z$ indicates forecast for particular occasion

Specifies conditional distribution of weather ($\Theta \mid Z = z$)

- **Climatology**

-- Unconditional probability distribution of Θ

(2) ECONOMIC VALUE OF FORECASTS

(2.1) OPTIMAL USE OF PROBABILITY FORECASTS

- **Criteria for Optimality**

-- Maximize expected return or minimize expected expense

-- Maximize expected utility (utility function u, payoff $u[w(a, \theta)]$)

Reflects risk preferences: risk neutral, risk aversion, risk taking

Expected value: Weighted average with respect to conditional probability distribution for future weather (as specified by forecast)

$$\max_a E_{\Theta|Z}\{u[w(a, \Theta)] \mid Z = z\}$$

(2.2) ECONOMIC VALUE OF PROBABILITY FORECASTS

- **Standard of Comparison**

-- e. g., climatological information

Other examples ("persistence", alternative forecasting system)

- **Concept of Value of Imperfect Information (VOI)**

-- Measure of how much better off decision maker is

(with vs. without forecasting system)

-- e. g., increase in expected return (or reduction in expected expense)

for forecasting system as compared to climatology

- **General Definition of VOI ("Demand Value")**

-- Maximum amount, measured in same units as those in which consequence of decision measured, that decision maker would be willing to exchange for information system

-- Formal Definition of Demand Value

$$E_Z\left(\max_a E_{\Theta|Z}\{u[w(a, \Theta) - \text{VOI}] \,\middle|\, Z\}\right) = \max_a E_\Theta\{u[w(a, \Theta)]\}$$

-- Consistent with concept of "contingent valuation" in economics

(3) COST-LOSS DECISION-MAKING MODEL

(3.1) EXPENSE MATRIX [$w(a, \theta)$]

Action	Weather Event: Adverse ($\Theta = 1$)	Weather Event: Not Adverse ($\Theta = 0$)
Protect ($a = 1$)	*C*	*C*
Do Not Protect ($a = 0$)	*L*	0

If take protective action, then incur cost *C*

If do *not* protect and adverse weather event occurs, then incur loss *L*

(3.2) OPTIMAL USE OF PROBABILITY FORECASTS

- **Decision-Making Criterion**

-- **Minimize expected expense**

- **Optimal Policy**

-- **Probability of adverse weather event p**

-- **Expected Expense (Protect): C**

-- **Expected Expense (Do Not Protect): $(1 - p)\ 0 + p\ L = p\ L$**

-- **So protect if $p > C / L$**

(3.3) ECONOMIC VALUE OF PROBABILITY FORECASTS

- **Climatology**

-- Climatological probability of adverse weather $p_A = \Pr\{\Theta = 1\}$

-- Minimal Expected Expense E_{CLIM}

$$E_{CLIM} = p_A L, \text{ if } p_A \leq C / L$$

$$E_{CLIM} = C, \text{ if } p_A > C / L$$

- **Forecasts (with Minimal Expected Expense** E_{FORE}**)**

-- Value of Forecasts: $E_{CLIM} - E_{FORE}$

(4) QUALITY-VALUE RELATIONSHIPS

(4.1) MODEL FOR PROBABILITY FORECASTING SYSTEM

- **Beta Distribution (for probability forecasts)**

-- Natural distribution for probabilities, $0 < p < 1$

-- Parameters r, s $(0 < r < \infty, 0 < s < \infty)$

-- Mean: $r / (r + s)$

-- Assume perfectly reliable probability forecasts

In particular, $p_A = r / (r + s)$

-- Climatology (i. e., no skill)

$r \rightarrow \infty, s \rightarrow \infty$

-- Perfect information

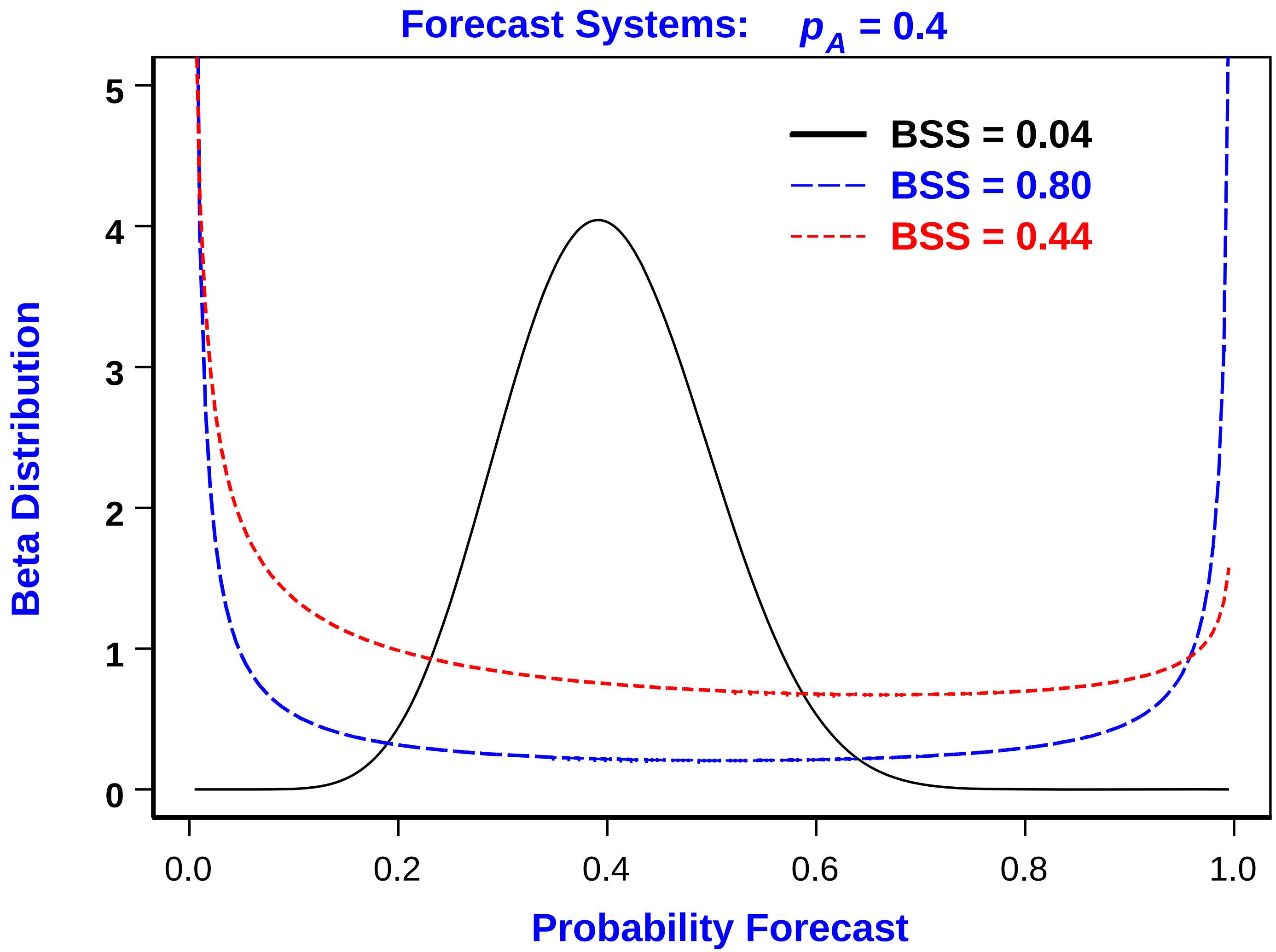
$r \rightarrow 0, s \rightarrow 0$

-- Brier skill score (BSS)

$$BSS = 1 / (r + s + 1)$$

Climatology: BSS = 0

Perfect information: BSS = 1


Forecast Systems: $p_A = 0.4$
BSS = 0.04
BSS = 0.80
BSS = 0.44
Beta Distribution
Probability Forecast
0
1
2
3
4
5
0.0
0.2
0.4
0.6
0.8
1.0

(4.2) EXAMPLE

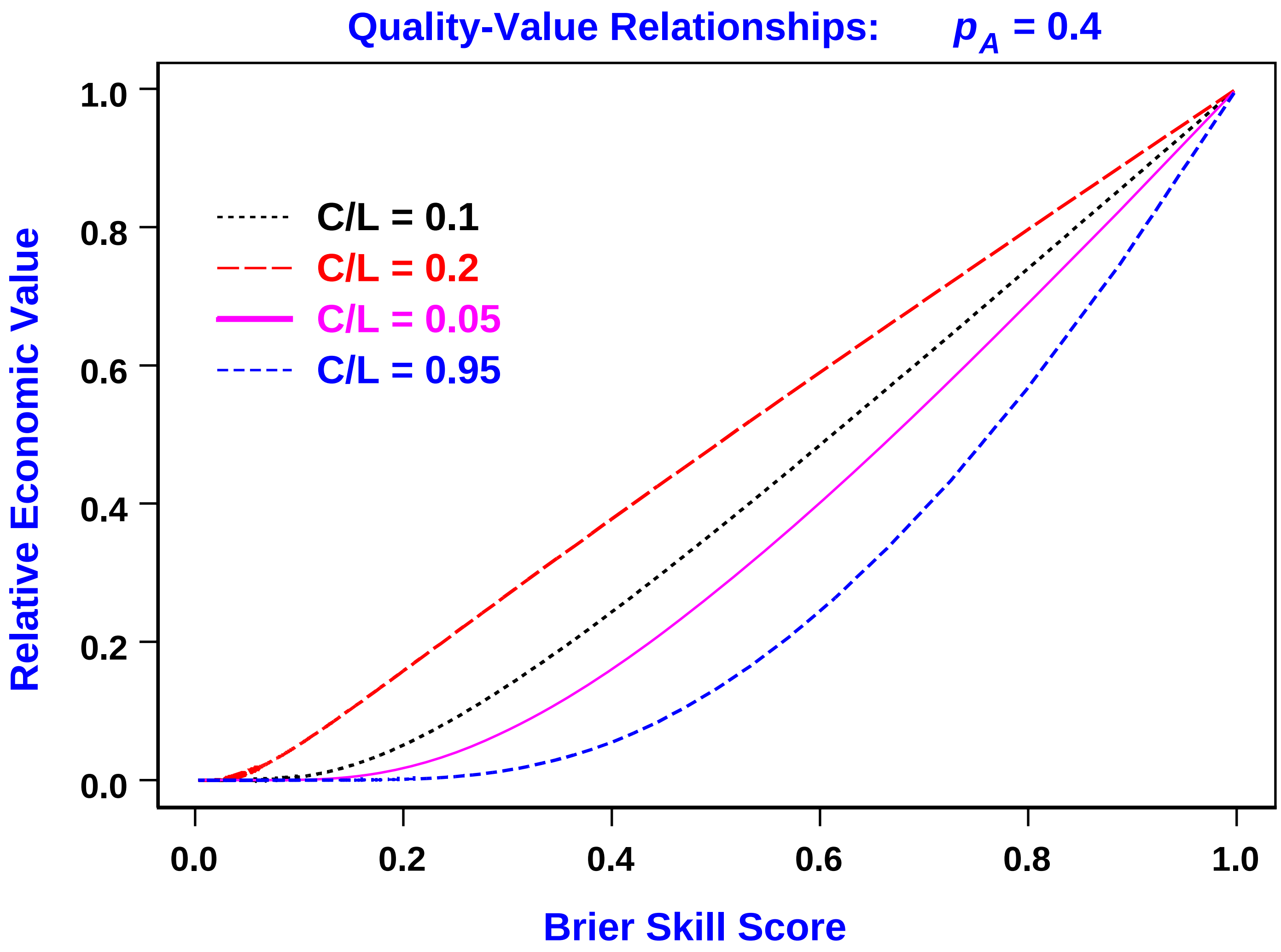
- **Cost-Loss Decision-Making Model**

-- Fix climatological prob. of adverse weather p_A

-- Fix cost-loss ratio C / L

-- Vary skill of forecasting system (through parameters r & s of beta distribution) from no skill (i. e., climatology) to perfect information

-- Examine how economic value of forecasting system changes (on relative scale from zero for climatology to one for perfect)


Quality-Value Relationships: p_A = 0.4
Relative Economic Value
Brier Skill Score
C/L = 0.1
C/L = 0.2
C/L = 0.05
C/L = 0.95
0.0
0.2
0.4
0.6
0.8
1.0

- **General Result**

-- Concept of "sufficiency"

Criterion for comparison of "quality" of two forecasting systems

-- Blackwell's Theorem

Quality-value relationships must be non-decreasing

(5) VALUATION PUZZLES

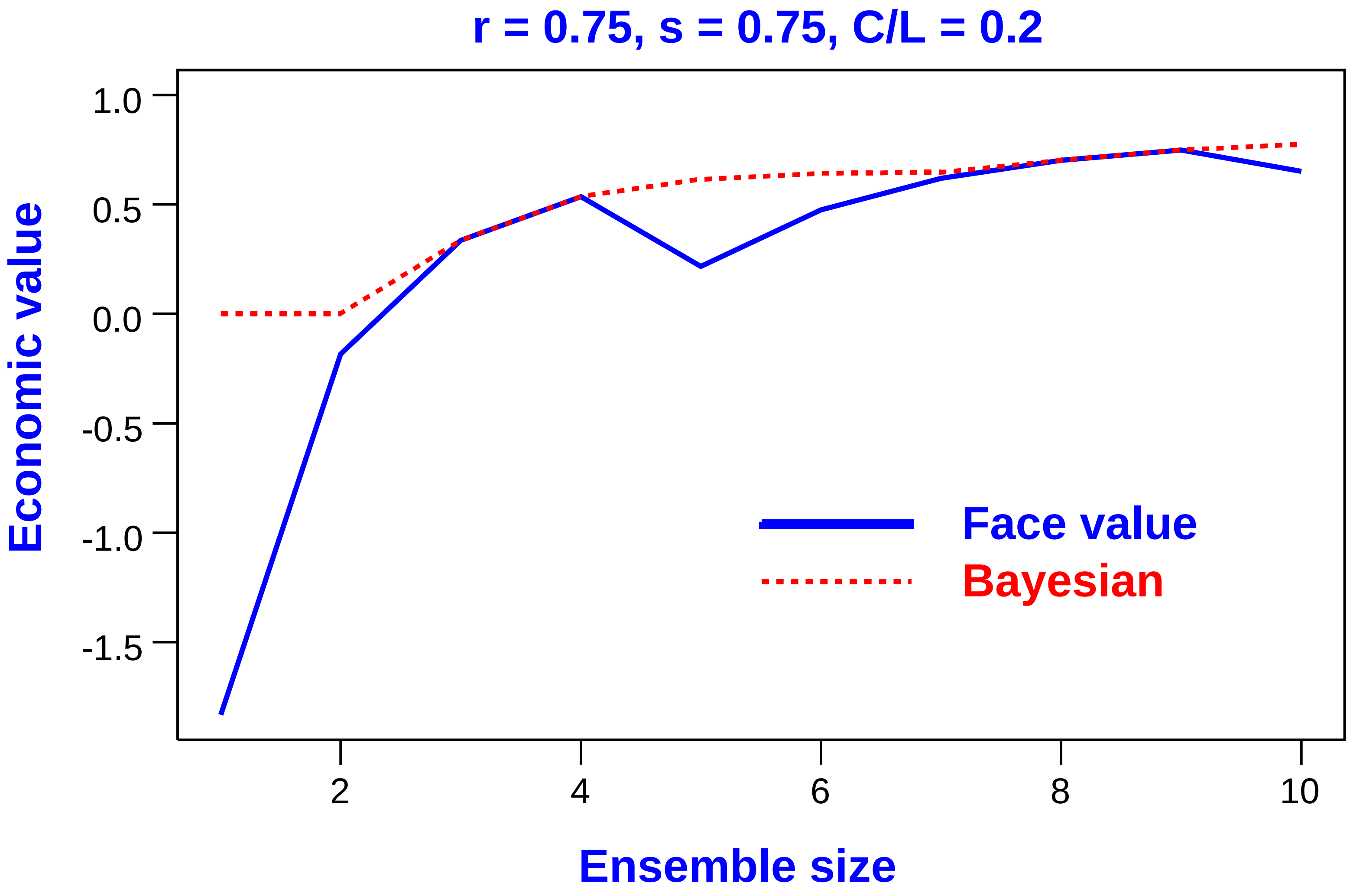
(5.1) QUALITY-VALUE REVERSALS

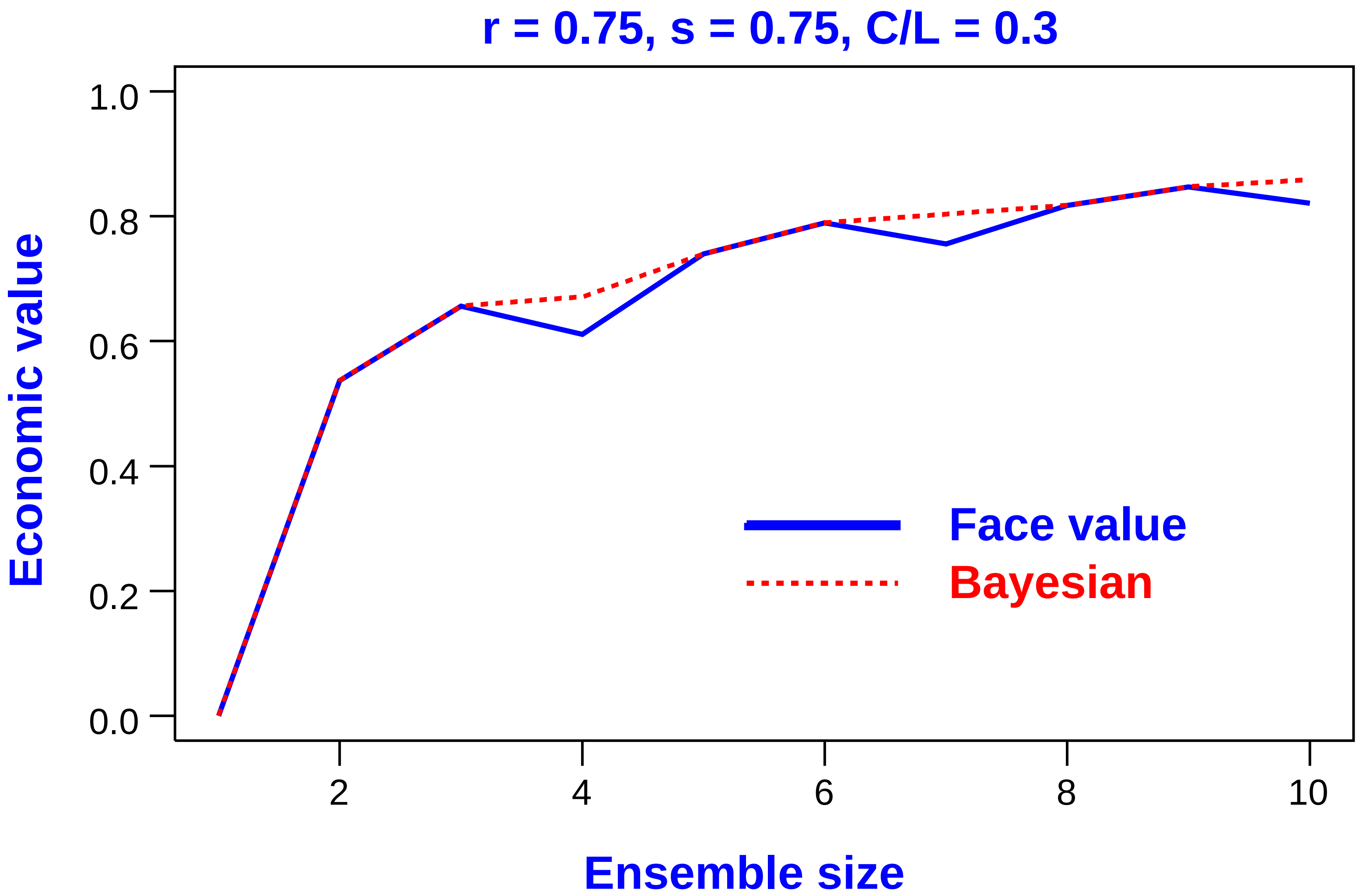
- **Cost-Loss Decision-Making Model**

-- Ensemble prediction system (perfect model)

Finite number of ensembles

Take ensembles at "face value"


r = 0.75, s = 0.75, C/L = 0.2
Economic value
1.0
0.5
0.0
-0.5
-1.0
-1.5
2
4
6
8
10
Ensemble size
Face value
Bayesian


r = 0.75, s = 0.75, C/L = 0.3
Economic value
1.0
0.8
0.6
0.4
0.2
0.0
2
4
6
8
10
Ensemble size
Face value
Bayesian

(5.2) DEARTH OF GENERAL RESULTS

- **Attributes of decision setting & decision maker**

-- **Flexibility**

-- **Risk aversion**

-- **Wealth**

-- **Prior uncertainty**

- **No monotonic relationship between information value and any of these attributes**

(6) RESOURCES

- **Recent Case studies**

-- Web site `www.isse.ucar.edu/HP_rick/esig.html`

Any published study in which economic value of weather or climate forecasts quantified

Tables listing case study attributes:

Structure of decision problem

Forecast characteristics

Information Valuation