UNCERTAINTY, OPTIMAL USE, AND ECONOMIC VALUE OF WEATHER FORECASTS

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Quotes

"If a man will begin with certainties he shall end in doubts; but if he will be content to begin with doubts he shall end in certainties."

Sir Francis Bacon

"You don't get points for predicting rain. You get points for building arks."

Lou Gerstner

(Former IBM CEO)

Outline

(1) Communicating Uncertainty

(2) Prototypical Decision-Making Model

(3) Economic Value of Forecasts

(4) Statistical Model for Probability Forecasts

(5) Quality / Value Curves

(6) Demonstration

(7) Resources

(1) Communicating Uncertainty

- Interpretation of Probability

-- "Frequentist"

Relative frequency (e. g., based on analogues in past)

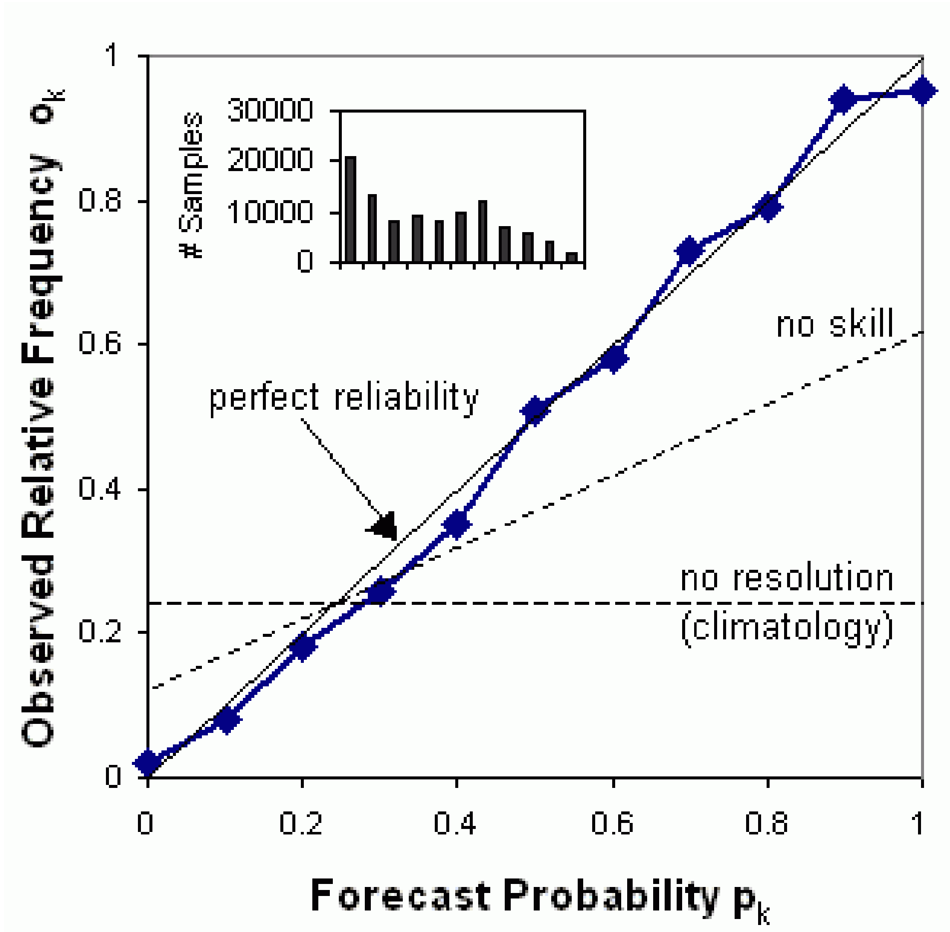
-- "Subjective"

Degree of belief (e. g., modify guidance based on own opinion)

-- "Propensity"

True underlying probability density function

- **Reliability Diagram**



- **Ambiguity about Forecasts**

  Fischhoff, 1994: "*A forecast is just the set of probabilities attached to a set of future events*"

-- **Ambiguity about probabilities**

  Qualitative expressions (e. g., "likely")

-- **Ambiguity about events**

  Murphy et al. (1980):

  Probability forecasts of precipitation occurrence

  (public confused about event, *not* probability per se)

- **AMS Statement (2002)**

  "Enhancing Weather Information with Probability Forecasts"

  `http://www.ametsoc.org/policy/enhancingwxprob_final.html`

-- **Quotes**

  "*The American Meteorological Society endorses probability forecasts and recommends their use be substantially increased.*"

  "*The quantification* [of uncertainty] *would almost certainly involve numerical probabilities*."

  "*Widespread use of probability forecasts will require significant efforts to educate the user in the definition of the event being forecast*."

“*New ways for displaying and communicating probabilistic information are needed*.”

“*Care must be taken that these forecasts are well calibrated*.”

“*Successful implementation and use of probability forecasts will require forecasters to understand user needs for this information and to be trained in how to best use the guidance produced by models to make probability forecasts*.”

- **NAS Report (2006)**

  “Completing the Forecast: Characterizing and Communicating Uncertainty for Better Decisions Using Weather and Climate Forecasts”

  `http://www.nap.edu/catalog/11699.html`

-- **Quotes**

  “. . . *no forecast is complete without a description of its uncertainty.*”

  “. . . *the information in deterministic forecasts is sometimes confusing or misleading, which can lead to poor decisions and undesirable outcomes.*”

“. . . *ongoing training of forecasters should expose them to the latest tools in these areas* [uncertainty and risk communication].”

”. . . *NWS has the responsibility to take a leading role in the transition to widespread effective incorporation of uncertainty information into hydrometeorological prediction.*”

“*NOAA should improve its product development process by collaborating with users and partners in the Enterprise from the onset and engaging and using social and behavioral expertise.*”

“*uncertainty champion*” (needed within NWS)

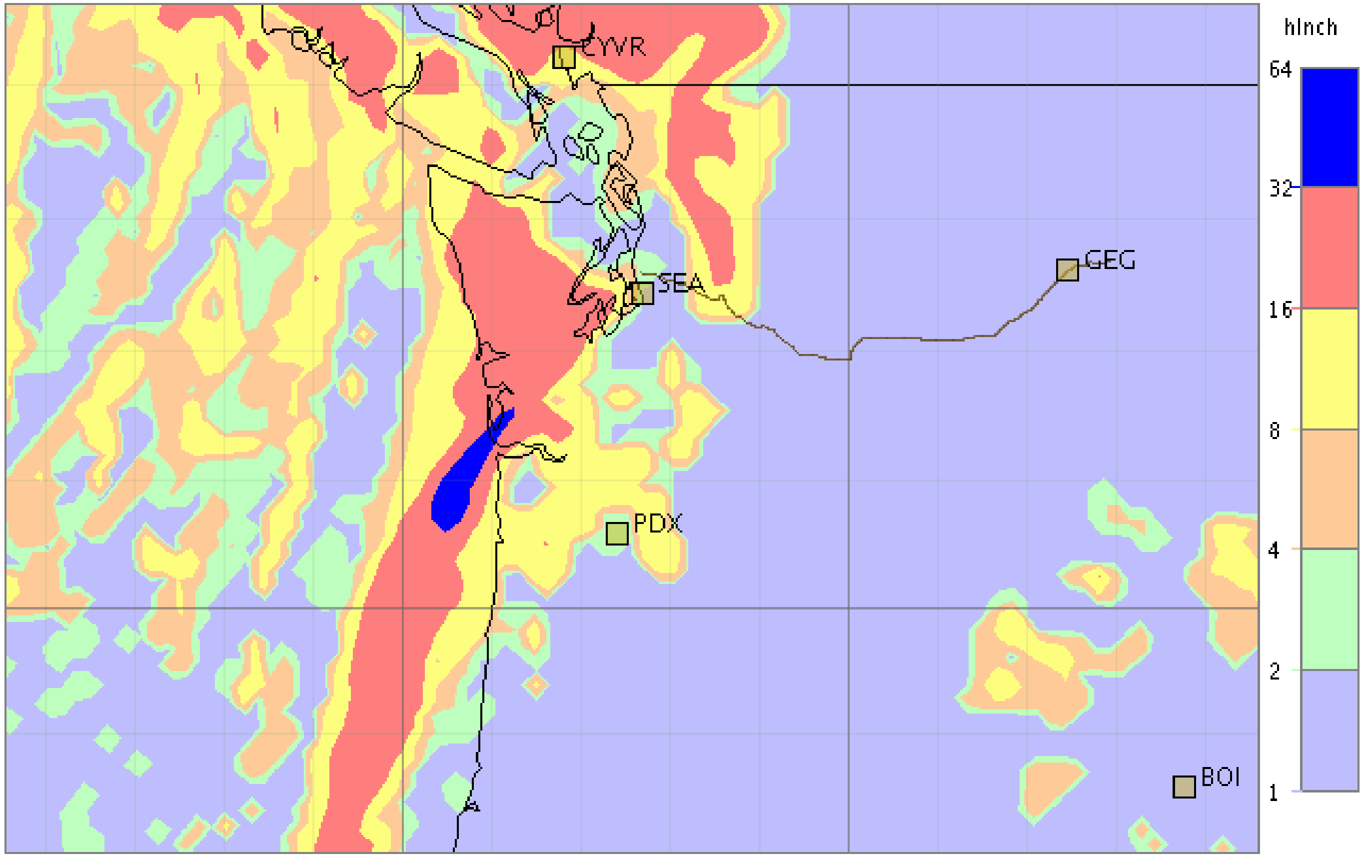
- **Projects on Communicating Forecast Uncertainty**

-- **Univ. of Washington**

"Integration and Visualization of Multi-Source Information for Mesoscale Meteorology: Statistical and Cognitive Approaches to Visualization of Uncertainty"

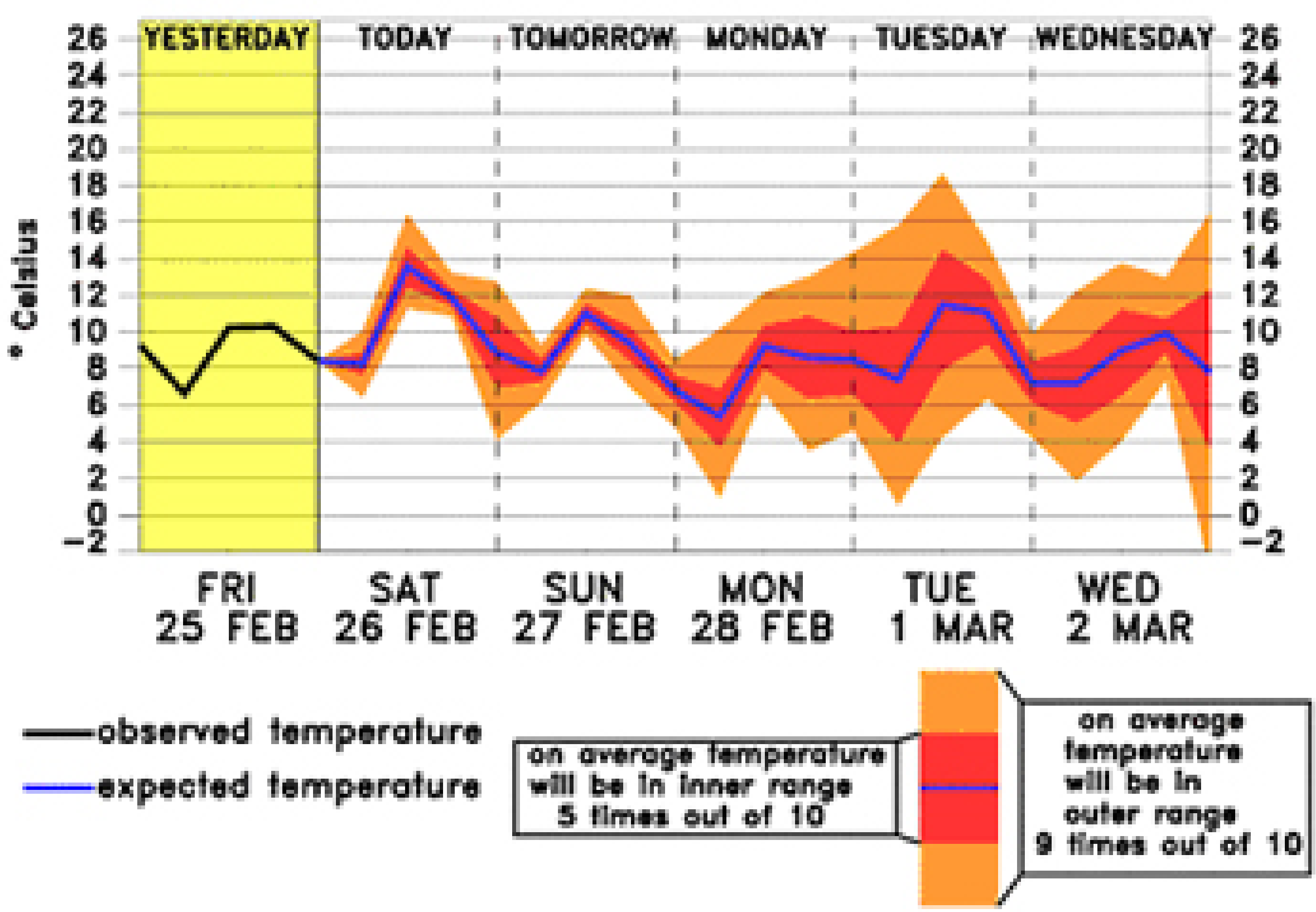
-- **Funded by ONR**

-- `http://www.stat.washington.edu/MURI/`



-- *“One time in ten the actual amount of precipitation is predicted to be greater than this amount”*

-- UK Met Office / Univ. Exeter / Pennsylvania State Univ.



(2) Prototypical Decision-Making Model

- **Cost-Loss Decision-Making Model**

-- Static decision-making structure

-- Only two possible actions (*a*)

Protect (*a* = 1) or do *not* protect (*a* = 0)

-- Only two possible states of weather (Θ)

Adverse (Θ = 1) or not adverse (Θ = 0)

-- Assume user expense minimizer

(equivalent to profit maximizer)

- **Expense Matrix**

Action	**Weather Event** **Adverse** ($\Theta = 1$)	 **Not Adverse** ($\Theta = 0$)
Protect ($a = 1$)	C	C
Do Not Protect ($a = 0$)	L	**0**

If take protective action, then incur cost C

If do *not* protect and adverse weather event occurs, then incur loss L

- **More Realistic Decision-Making Models**

-- Dynamic (*not* static)

-- Risk aversion (*not* profit maximizer)

-- Complexity

More than two actions

(e. g., long-term strategic, short-term tactical)

More than two weather states

(3) Economic Value of Forecasts

- **Optimal use of forecasts**

-- Given forecast probability of adverse weather event p

Expected Expense (*Protect*): C

Expected Expense (*Do Not Protect*): $(1 - p)\ 0 + p\ L = p\ L$

So protect if $p > C/L$

- **Economic Value of Forecast System**

-- Climatology

Climatological probability of adverse weather

$$p_A = \Pr\{\Theta = 1\}$$

Minimal Expected Expense E_{CLIM}

$$E_{\mathrm{CLIM}} = p_A L \qquad \text{if } p_A \leq C/L$$

$$E_{\mathrm{CLIM}} = C \qquad \text{if } p_A > C/L$$

-- Forecasts (with Minimal Expected Expense E_{FORE})

$\Pr\{\Theta = 1\}$ **varies**

(i. e., conditional probability of adverse weather)

-- **Attributes of probability forecasts (besides reliabililty)**

"Resolution" or "sharpness"

-- **Value of information (VOI)**

Reduction in expected expense (relative to climatology):

$$VOI = E_{CLIM} - E_{FORE}$$

Interpretation:

Willingness-to-pay (contingent valuation)

"Demand value"

(4) Statistical Model for Probability Forecasts

- **Statistical Model for Generating Probability Forecasts**
  (e. g., from perfect numerical weather prediction system)

-- **Beta Distribution (for probability forecast p)**

Natural distribution for probabilities, $0 < p < 1$

Parameters r, s $(0 < r < \infty, 0 < s < \infty)$

Mean: $r / (r + s)$

Mode: $(r - 1) / (r + s - 2)$, $r \,\&\, s > 1$

- **Assume perfectly reliable probability forecasts**

  Then $p_A = r / (r + s)$

  So constrain $s = r\,[(1 - p_A) / p_A]$

  (i. e., only one free parameter r **)**

-- Climatology (i. e., no skill)

$r \to \infty,\ s \to \infty$ **(holding** p_A **constant)**

-- Perfect information

$r \to 0,\ s \to 0$ **(holding** p_A **constant)**

- Brier score (BS)

$$BS = E[(\Theta - p)^2]$$

-- Brier skill score (BSS)

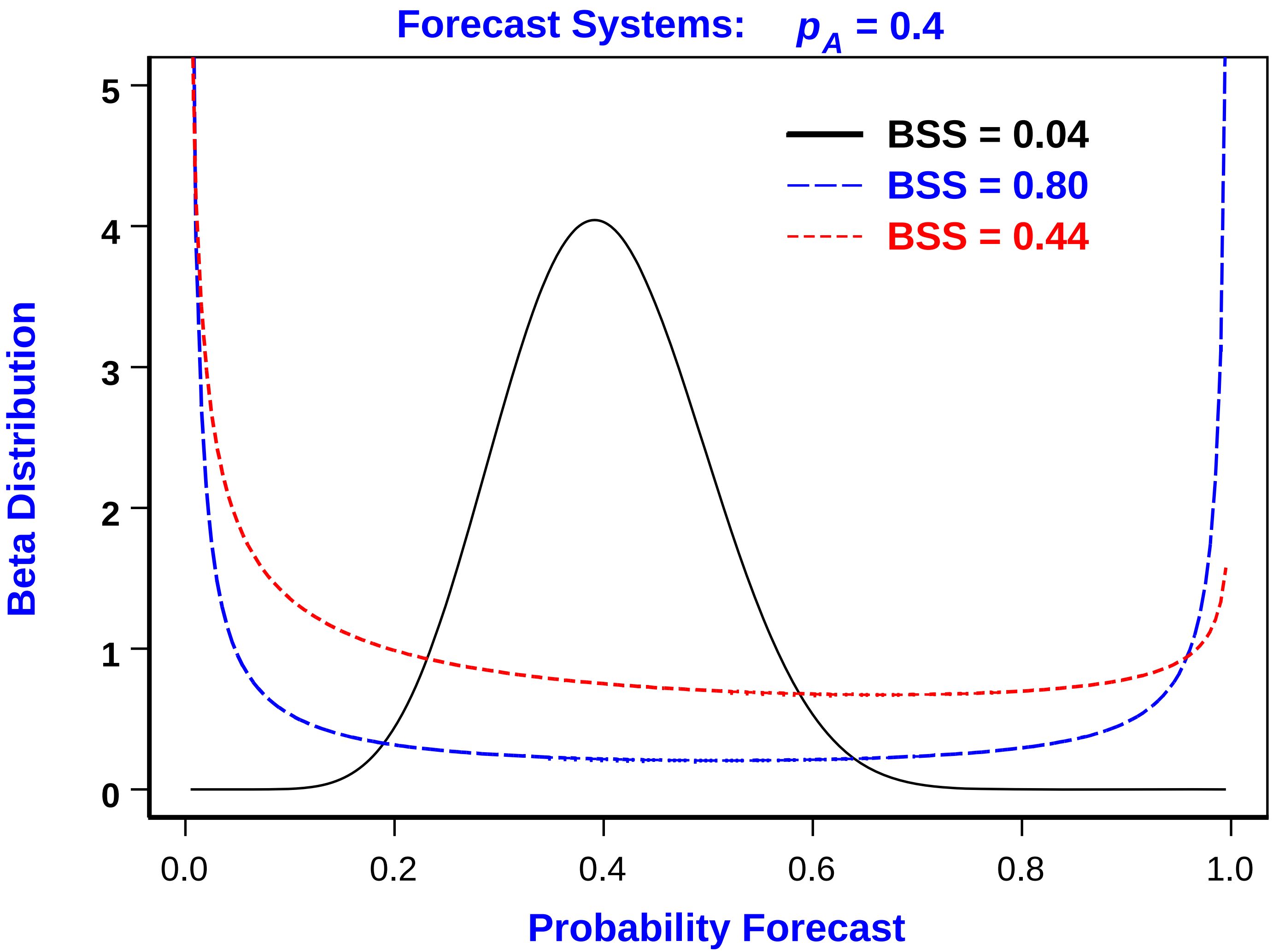
$$BSS = 1 - [BS / Var(\Theta)], \quad 0 \leq BSS \leq 1$$

Climatology: $BSS = 0$

Perfect information: $BSS = 1$

Model based on beta distribution:

$$BSS = 1 / (r + s + 1)$$



Forecast Systems: p_A = 0.4
BSS = 0.04
BSS = 0.80
BSS = 0.44
Beta Distribution
0
1
2
3
4
5
0.0
0.2
0.4
0.6
0.8
1.0
Probability Forecast

(5) Quality / Value Curves

- **Cost-Loss Decision-Making Model**

-- Fix climatological prob. of adverse weather p_A

Fix cost-loss ratio C/L **(i. e., specific user)**

- **Perfectly Reliable Probability Forecasting System**

-- e. g., perfect numerical weather prediction model with infinite ensemble size

-- **Vary skill of forecasting system (through parameter *r* of beta distribution) from no skill (i. e., climatology) to perfect information**

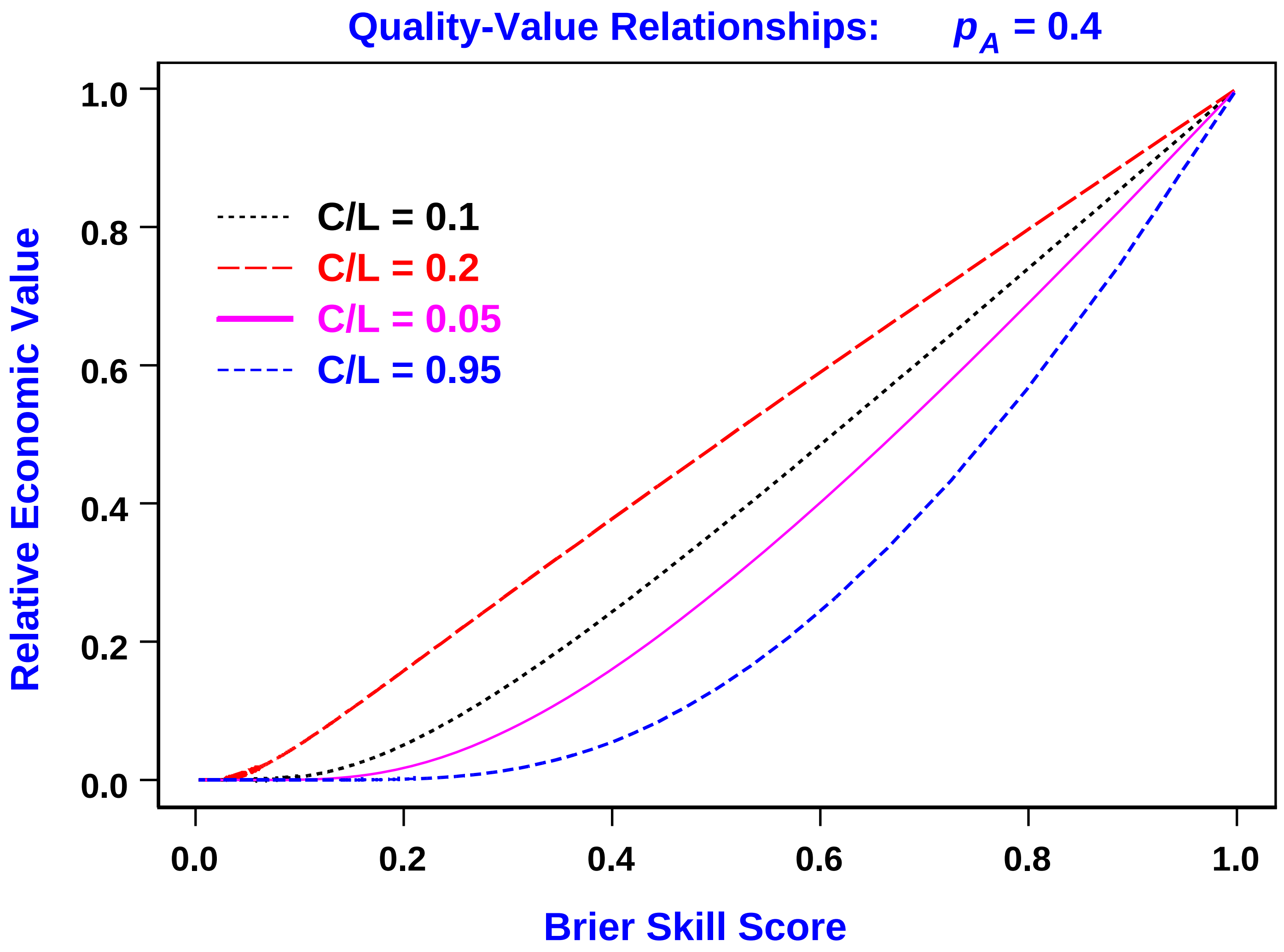
-- **Examine how economic value of forecasting system changes (on relative scale from zero for climatology to one for perfect)**

-- **Decision-theoretic framework**

Economic value must be nonnegative

Economic value of forecasts cannot decrease as quality of forecasting system increases

Could be threshold in quality below which no economic value


Quality-Value Relationships: p_A = 0.4
Relative Economic Value
Brier Skill Score
C/L = 0.1
C/L = 0.2
C/L = 0.05
C/L = 0.95
0.0
0.2
0.4
0.6
0.8
1.0

- **Ensemble Prediction**

-- **Still perfect NWP, but now only finite number *m* of ensembles available**

Generate these ensembles using underlying forecast probability *p* from beta distribution

-- **Usual approach**

Take ensembles at "face value":

Suppose event occurs for *k* out *m* ensembles

Estimate forecast probability as

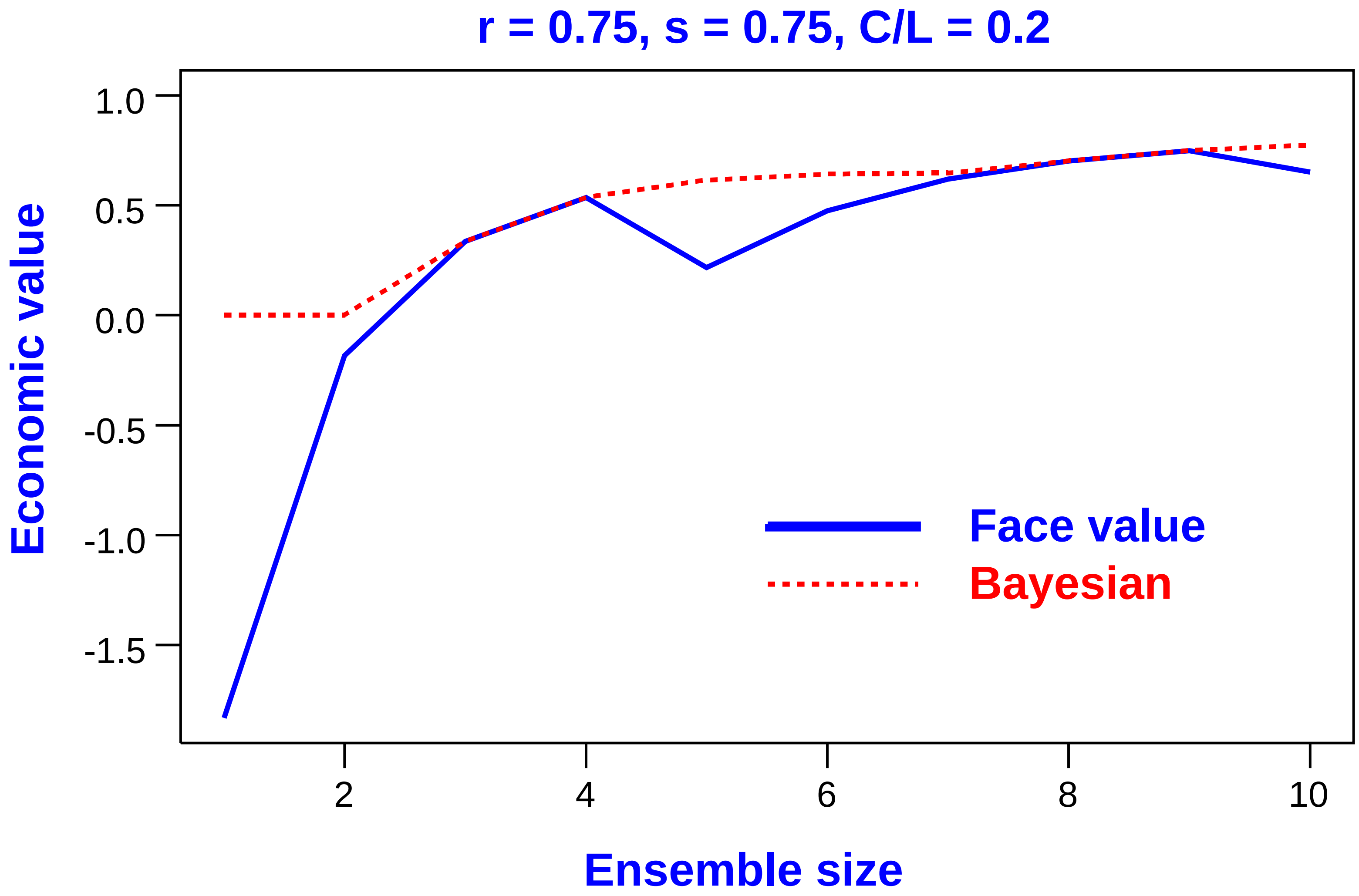
$$\hat{p} = k/m$$

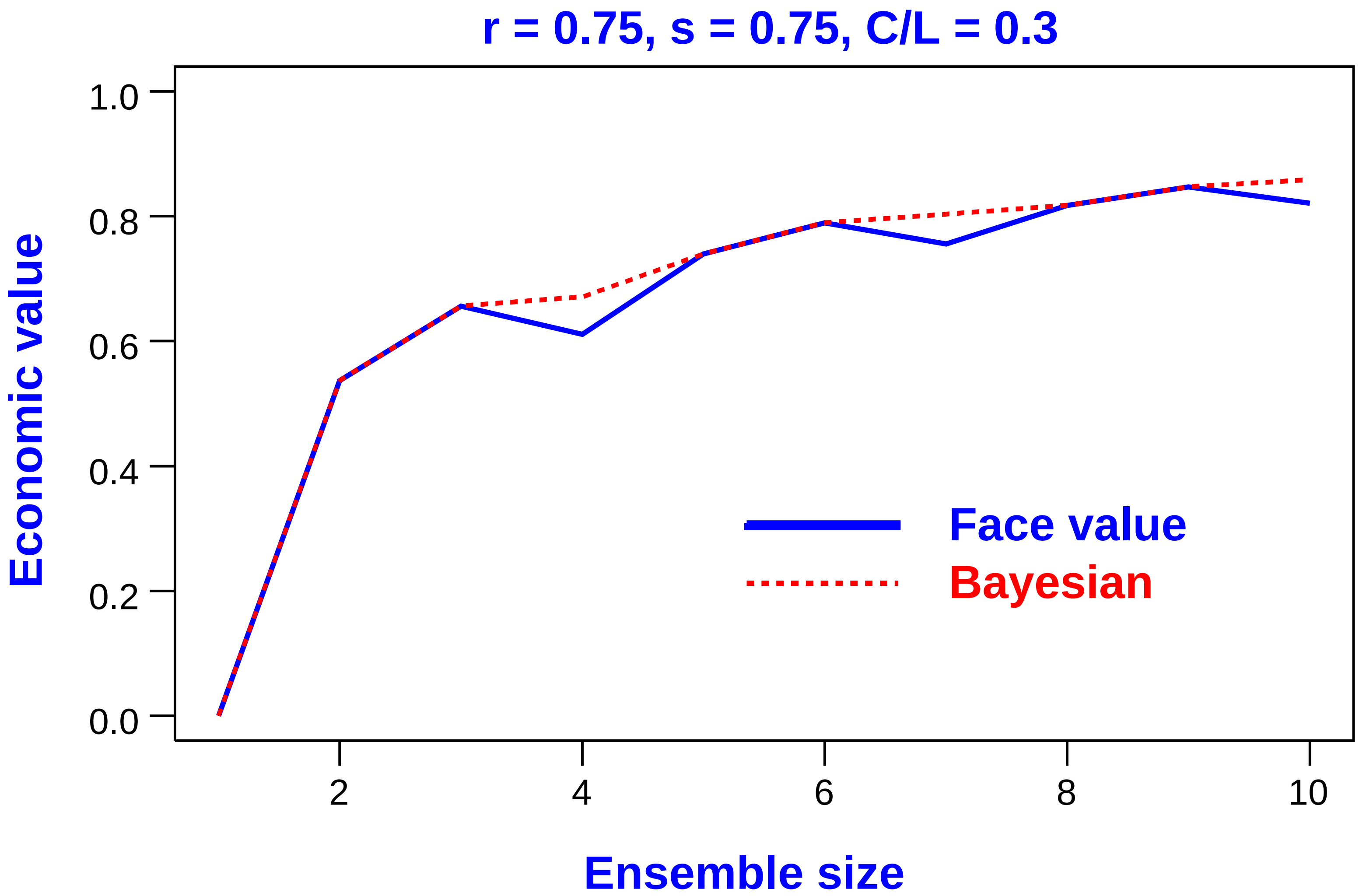
-- **Alternative Bayesian approach**

(in which uncertainty in estimating *p* also taken in account, like smoothing / re-calibration)

-- **Cost-Loss Decision-Making Model**

Usual prototypical model for valuing ensemble forecasts


r = 0.75, s = 0.75, C/L = 0.2
1.0
0.5
0.0
-0.5
-1.0
-1.5
Economic value
Face value
Bayesian
2
4
6
8
10
Ensemble size


r = 0.75, s = 0.75, C/L = 0.3
Economic value
1.0
0.8
0.6
0.4
0.2
0.0
2
4
6
8
10
Ensemble size
Face value
Bayesian

(6) Demonstration

- **Statistical Programming Language R**

-- The R Project for Statistical Computing

Open source: `http://www.r-project.org/`

-- Simulate from beta distribution, Bernoulli distribution, binomial distribution, etc.

-- Possible interpretations

(i) Repetitive decision making process (average outcomes)

(ii) Single future occasion (for which *not* even forecast yet available)

- **Simulation Example (*Reliable Probability Forecasts*)**

-- User (parameters C & L)

Say $C = 0.2$, $L = 1$

-- Climatology (parameter p_A)

Say $p_A = 1/3$

-- Forecasting system (parameter r of beta dist.)

Say $r = 0.5$ (so $s = 1$)

BSS = 1 / 2.5 = 0.4

- **First Occasion**

(i) Generate probability forecast from beta dist.

Random number generator (beta dist. with $r = 0.5$ & $s = 1$): $p \approx 0.09$

(ii) Select action

$p = 0.09 < 0.2 = C/L$ (so do *not* take protective action)

(iii) Generate actual weather event

Random number generator (Bernoulli dist. with prob. $p \approx 0.09$): $\Theta = 0$

(iv) Outcome (cost C or loss L)

So incur no cost or loss

- **Second Occasion**

(i) Generate probability forecast from beta dist.

Random number generator (beta dist. with $r = 0.5$ & $s = 1$): $p \approx 0.52$

(ii) Select action

$p = 0.52 > 0.2 = C/L$ (so take protective action)

(iii) Generate actual weather event

Random number generator (Bernoulli dist. with prob. $p \approx 0.52$): $\Theta = 0$

(iv) Outcome (cost C or loss L)

So incur cost $C = 0.2$

- Third Occasion

(i) Generate probability forecast from beta dist.

Random number generator (beta dist. with $r = 0.5$ & $s = 1$): $p \approx 0.47$

(ii) Select action

$p = 0.47 > 0.2 = C/L$ (so take protective action)

(iii) Generate actual weather event

Random number generator (Bernoulli dist. with prob. $p \approx 0.47$): $\Theta = 1$

(iv) Outcome (cost C or loss L)

So incur cost $C = 0.2$

- **Repeat Process Indefinitely**

-- Average expense incurred over all occasions*

(i. e., estimate of E_{FORE})

-- Expected expense with climatology

Because $p_A = 1/3 > 0.2 = C/L$

should always protect with climatological information:

So $E_{CLIM} = C = 0.2$

***Note*: Actually can compute this expected expense exactly**

(i. e., without resort to simulation;

see Katz & Ehrendorfer, *Weather and Forecasting*, 2006)

- **Simulation Example** (***Ensembles from Perfect NWP Model***)

-- Simulate probability forecast *p* (as before from beta distribution)

-- Given *p*, simulate *m* ensembles from binomial distribution with parameters *m* & *p* (i. e., *m* Bernoulli trials)

-- Simulate actual weather as (*m* + 1)th Bernoulli trial with same prob. *p* (i. e., view actual weather as another independent ensemble*)

***Note*: Actual weather is *conditionally* independent of other ensembles, but *unconditionally* dependent**

(Because of common probability forecast *p*)

(7) Resources

- **Conferences**

  Systems approach / end-to-end / Integration

-- Monterey (Jan. 2007: "End-to-end integration from atmospheric science through operational forecast end-users")

`http://wikidev.nps.edu/Weather/index.php/Main_Page`

-- UK Met Office (June 2007: "Meteorology Meets Social Science: Risk, Forecast and Decision")

`http://people.ex.ac.uk/trkaplan/met/conference.html`

- **Recent Case Studies of Economic Value**

-- **www.isse.ucar.edu/HP_rick/esig.html**

Any published prescriptive study in which economic value of weather or climate forecasts quantified

Earlier studies reviewed in chapter by Dan Wilks (*Economic Value of Weather and Climate Forecasts*, edited by Katz & Murphy, Cambridge Univ. Press)

-- **Tables listing case study attributes**

Structure of decision problem

Forecast characteristics

Information Valuation

-- **Sectors**

Agriculture (by far most studies)

Energy

Fishery

Transportation

-- **Forecast variables**

El Niño-Southern Oscillation (ENSO) phenomenon (most attention)

-- **Example table**

Study	Solow, A.R., Adams, R.F., Bryant, K.J., Legler, D.M., O'Brien, J.J., McCarl, B.A., Nayda, W., and Weiher, R. (1998). The value of improved ENSO prediction to U.S. agriculture. *Climatic Change*, 39, 47-60.		
	Structure of Decision Problem	**Forecast Characteristics**	**Information Valuation**
	Decision: allocations among various crops Dynamics: no	Time Scale: annual Predictand: El Nino Format: categorical Type: realistic, derived Quality Changes: yes	Baselines: climatological and persistence VOI, imperfect: $240-$266 million/year (1995 US $) VOI, perfect: $323 million/year (1995 US $) Risk Treatment: expected value