Self-Organised Criticality and the Atmospheric Sciences: selected review, new findings and future directions

Suraje Dessai
Department of Environmental Studies
University of Colorado
Boulder, CO 80309-0397, USA
&
School of Environmental Sciences
University of East Anglia
Norwich NR4 7TJ, UK
S.Dessai@uea.ac.uk

Martin E. Walter
Department of Mathematics
University of Colorado
Boulder, CO 80309-0395, USA
Martin.Walter@colorado.edu

Abstract
Self-organised criticality (SOC) has been considered by some the foremost candidate for a universal theory of complexity. A review of past studies of SOC in the context of the atmospheric sciences, an area of relative neglect, is conducted. Using empirical data from extreme weather and climate events (hurricanes, windstorms and tornadoes) it is shown that these atmospheric phenomena display SOC behaviour.

1. Introduction

The concept of 'self-organised criticality' (SOC) was introduced by Bak *et al.* (1987) to explain the occurrence of power laws in nature. In its relatively short history SOC patterns have been found in a myriad of fields, ranging from earthquakes (Hainzl *et al.* 1999) to music (Voss and Clark 1978) passing through wars (Roberts and Turcotte 1999), economics (Scheinkman and Woodford 1994) and forest fires (Drossel and Schwable 1992).[1] Long-range spatiotemporal correlations, identified by Bak *et al.* (1987) as SOC are signatures of deterministic chaos, which have been considered a new way of viewing nature. SOC systems are characterised by power law distributions of certain events (hurricanes, windstorms, quakes, etc.). Although large power events are comparatively rare, events can and do happen on all scales, with no different mechanism needed to explain the rare large events than that which explains the smaller, more common ones (Bak, 1996).

In the atmospheric sciences there has been little application of what some have considered the leading candidate for a unified theory of complexity. Meteorologists and

[1] These references are only examples of the application of SOC. Sand pile and earthquake models have been the most widely used applications.

climatologists, the main audience of this paper, have largely ignored SOC. In his review of complexity and climate, Rind (1999) concludes climate, like weather will likely always be complex: "*determinism in the midst of chaos, unpredictability in the midst of understanding*." He warns that it is still not known if complexity is relevant to climate science. We argue that there is enough evidence, from past research, which we review, and from new research, which we present, to show that complexity and its theory of SOC have considerable potential to increase our understanding of the atmospheric sciences.

2. A selected review

We suspect theories of complexity, such as SOC, have been underrepresented in the atmospheric sciences because of their "soft science" character. Atmospheric sciences have historically developed from centuries of advancement in the hard sciences, such as physics, mathematics and chemistry, etc. It would have been unlikely to see a quick transition from the classical reductionist and reproducible science approach towards an abstract, holistic and probabilistic complex science. Proof of this is the fact that only a small number of scientists have cited the few applications of these theories in the atmospheric sciences.[2] A short review of these applications follows.

Selvam (2000) and colleagues from the Indian Institute of Tropical Meteorology are probably at the forefront of SOC and the atmospheric sciences. They have published considerably in this field. Using a cell dynamical system model for atmospheric flows Selvam (1990) showed that it exhibits long-range spatial and temporal correlations, signatures of SOC. Using continuous periodogram spectral analysis of rainfall time-series (Selvam *et al.* 1992), global surface (air and sea) temperature time-series (Selvam and Joshi 1995), and annual and seasonal mean global surface pressure time-series (Selvam *et al.* 1996), it was shown that the power spectra follows the universal inverse power-law form of the statistical normal distribution. Selvam *et al.* (1996) have argued that the inverse power-law form for power spectra is ubiquitous to real-world dynamical systems and is identified as a signature of SOC or deterministic chaos. Joshi and Selvam (1999) have also recently identified SOC signatures in atmospheric low frequency variability.

Focusing particularly on rain and clouds Lovejoy and his group established the applicability of fractals in meteorology, by showing that cloud and rain areas project on the Earth along shapes whose boundaries are fractal curves, and that the temporal and spatial structure of rain is rife with hyperbolically distributed features (Lovejoy 1982; Lovejoy and Mandelbrot 1985; Lovejoy and Schertzer 1986). Since then, there have been a series of papers investigating modelled and empirical results for rain and clouds (Lovejoy and Schertzer 1990; Hubert *et al.* 1993; Tessier *et al.* 1993). Schertzer and Lovejoy (1997) conclude that the multifractal approach yields a convenient framework for the analysis and simulation of highly nonlinear meteorological fields over a wide range of scales and intensities.

[2] Another possibility for the neglect of SOC in the atmospheric sciences is the increased funding of applied atmospheric sciences (e.g. climate change research) *vis-à-vis* the decreased funding of "basic" research, i.e., according to Byerley (1995) research to increase knowledge; to answer a scientific as opposed to a practical question.

Besides the work of Selvam and Lovejoy groups' we found virtually no more studies in this area, with exception of Vattay and Harnos (1994), who demonstrated that daily average air humidity fluctuations manifested SOC behaviour. In light of the relatively small number of research efforts in the area of SOC and atmospheric science, we present some of our research results in this new area.

3. Recent findings

The main purpose of this section is to test the ideas of SOC using empirical data of meteorological and climatological phenomenon.

3.1 The general atmospheric circulation

We base this approach upon Koppány's (1975) efforts to estimate the period needed for the generation or transformation of the global atmospheric circulation. We plotted his estimates of the duration of atmospheric systems against their kinetic energy in a log-log graph (Figure 1). Similar figures have been reproduced in some undergraduate meteorology manuals (e.g. Barry and Chorley, 1987), but exclusively to compare the forces of nature against certain human activities. However, what Figure 1 shows is that atmospheric motion systems exhibit self-similar behaviour with a power-law of the form $f(x) = cx^B$, where *c* and *B* are constants. We find this power law to be:

$$f(x) = 2248x^{2.5553}$$

The fact that atmospheric motion systems have a fractional exponent, could, as suggested by Schroeder (1991), have a link with rather different situations (such as melting or magnetism, for instance) with the same exponent, providing a hint to similar underlying "universal" mechanics. This seems something worth investigating.

3.2 Extreme weather and climate events

Weather and climate extremes can have detrimental impacts upon society and ecosystems. Walter (1997) hypothesised that "the distribution of the number of extreme events versus the logarithm of their power is a geometric distribution." To our knowledge this hypothesis has not yet been demonstrated with empirical evidence. In the next subsection we show that Walter's hypothesis is true for most of the analysed weather and climate extremes.

3.2.1 Hurricanes

Hurricanes are intense circular vortices with sustained surface winds of 64 Kts (74 mph, 33 m/s) or greater, spiralling around a low pressure centre. These extreme events start over the tropical oceans, between about 7 and 15º latitude, when sea-surface temperatures are greater than 27º C. They can be measured with the Saffir-Simpson Scale, which is a 1-5 rating based on the hurricane's present intensity (Table 1). Wind speed is the determining factor in the scale, as storm surge values are highly dependent on the slope of the continental shelf in the landfall region (Landsea, 2000).

Table 1. The Saffir-Simpson Hurricane Scale. Source: Landsea (2000)

Saffir-Simpson Scale						
Saffir-Simpson Category	Maximum sustained wind speed			Minimum surface pressure	Storm surge	
	mph	m/s	kts	mb	ft	m
1	74-95	33-42	64-82	greater than 980	3-5	1.0-1.7
2	96-110	43-49	83-95	979-965	6-8	1.8-2.6
3	111-130	50-58	96-113	964-945	9-12	2.7-3.8
4	131-155	59-69	114-135	944-920	13-18	3.9-5.6
5	156+	70+	136+	less than 920	19+	5.7+

We use data from NOAA’s Hurricane Research Division (2000) from 1950-95 for Atlantic basin hurricanes and East Pacific basin hurricanes.[3] Figure 2 shows the empirical evidence that hurricanes, like earthquakes, follow power laws. Atlantic basin hurricanes follow the inverse power-law $f(x) = 8.476x^{-2.2961}$ and Pacific basin hurricanes $f(x) = 5.6359x^{-1.0362}$.

3.2.2 Windstorms

Convective winds of non-tornadic thunderstorms can cause damage to infrastructure (e.g. power lines) and ecosystems (by knocking down trees and increasing the mortality of individual trees). Recent events in Europe (particularly in France and Switzerland) have shown the power of windstorms in decimating entire forests. We use data from NOAA’s Storm Prediction Center (2000) from 1955-95 for the US. Figure 3 shows the distribution of high winds, which follows a simple inverse power-law form. With an r-squared of 0.98, windstorms in the US follow the power law $f(x) = 10^{24} x^{-11.873}$.

3.2.3 Tornadoes

A tornado is a small mass of air (whirlwind) that spins rapidly about an almost vertical axis and forms a funnel cloud that contacts the ground; appears as a pendant form of a cumulonimbus cloud and is potentially the most destructive of all weather systems

[3] It should be noted that there are well known biases in the East Pacific tropical cyclone data base. There is a decided increase in reported storms with the advent of weather satellites. Very few of the wind values are based on aircraft reconnaisence, most are rough estimates from satellite pictures (HRD, 2000).

(Geer 1996). The Fujita Scale is used to rate the intensity of a tornado by examining the damage caused by the tornado after it has passed over a man-made structure (Table 2).

Table 2. The Fujita Tornado Scale. Source: Tornado Project (2000)

F-Scale	F0	F1	F2	F3	F4	F5
Windspeed (mph)	40-72	73-112	113-157	158-206	207-260	261-318

This method of indirect measurement of intensity is very subjective, but has been widely used because of its practicality. One of the drawbacks of this scale, which we are most concerned here, is the underreporting of weak tornados, which has been estimated to be an additional 1000 weak tornadoes per year (Tornado Project 2000) or according to Brooks (2000), the number of F1 tornadoes should be 27% of the number of F0 tornadoes. Using data from NOAA's Storm Prediction Center (2000) from 1955-95 for the US we show that empirical data for tornado intensity does not appear to show a SOC signature (see Figure 4). This is largely due to the underreporting, hence using a dashed line we have speculated how tornadoes might "truly" be distributed. Further investigation is needed to understand the distribution of tornadoes.

3.3 Others

The last example we present considers applied meteorology, where we also think signatures of deterministic chaos abound and have yet to be explored. Forest fires have already been shown to exhibit power-law frequency-area statistics over many orders of magnitude (Drossel and Schwable 1992; Malamud *et al.* 1998). Borrowing this example from Viegas (1998), Figure 5 shows the burned area of forests in Portugal as a function of precipitation (from June to September), which follows a power-law frequency-area behavior, a signature of SOC.

4. Concluding remarks and future prospects

We believe many more examples of SOC behaviour exist in the atmospheric sciences. Table 3 summarises our recent findings in the form of the power-law exponent (*B*) and how well the empirical data compared with the theory of SOC (r-squared).

Table 3. Exponent and r-squared of the tested atmospheric phenomena.

	Exponent	R-squared
Atlantic hurricanes	2.2961	0.819
Pacific hurricanes	1.0362	0.7405
Windstorms	11.873	0.98
Tornadoes	2.7215	0.8478
Forest fires and precipitation	0.6551	0.4736

The tornado values are based upon the aggregation of the Fujita Scale into weak (F0-F1), strong (F2-F3) and violent (F4-F5) tornadoes. Is there a self-organising pattern within these phenomena, like with the general circulation? The exponents vary, but some of them might be relevant in other situations, e.g. Lovejoy and Schertzer (1986) found an exponent of 11/5 for vertical wind, similar to our Atlantic hurricane values, which might indicate a common pattern for these two phenomena. Investigating these common patterns seems worthwhile. Except for the forest fire and precipitation results, most modelled results are in accordance with the empirical data (some more than others). Complexity and its theory of SOC have valuable applications in the atmospheric sciences. We have shown that modelled results are close to empirical data for most extreme weather and climate events. Many other applications are foreseeable within the atmospheric sciences. This is work we intend to pursue in the future. We hope this paper will attract more atmospheric scientists to apply SOC to their studies. Complexity and SOC will most likely become the fundamental sciences of the 21st century. It is quite likely that our understanding of atmospheric systems will benefit from the interaction of the evolving sciences of SOC and complexity.

Acknowledgments
We would like to thank Domingos Xavier Viegas for providing us Figure 5.

References

Bak, P. (1996) *How nature works: the science of self-organized criticality*. Copernicus, New York, 212 pp.

Bak, P., Tang, C., and Wiesenfeld, K. (1987) Self-organized criticality: an explanation of 1/f noise. *Phys. Rev. Lett.*, **59**, 381-384.

Barry, R. G., and Chorley, R. J. (1987) *Atmosphere, Weather and Climate* (5th ed.). Methuen, New York, 460 pp.

Brooks, H. (2000) Tornado Reports by Damage Class http://www.nssl.noaa.gov/~brooks/worldfscale.html, view on March 12, 2000.

Byerley, Jr., R. (1995) U.S. science in a changing context: A perspective. *Reviews of Geophysics*, Supplement, A1-A16.

Drossel, B., and Schwable, F. (1992) Self-Organized Criticality in a Forest-Fire Model, *Phys. Rev. Lett.*, **69**, 1629.

Geer, I. W. (1996) *Glossary of Weather and Climate*. Amer. Meteor. Soc., 272 pp.

Hainzl, S., Zoller, G., and Kurths, J. (1999) Self-organized criticality model for earthquakes: Quiescence, foreshocks and aftershocks. *International Journal of Bifurcation and Chaos*, **9**, 2249-2255.

Hubert, P., Tessier, Y., Lovejoy, S., Schertzer, D., Schmitt, F., Ladoy, P., Carbonnel, J. P., Violette, S., and Desurosne, I. (1993) Multifractals and extreme rainfall events. *Geographical Research Letters*, **20**, 931-934.

Hurricane Research Division – NOAA (2000) Hurricane Info by Basin Page http://www.aoml.noaa.gov/hrd/basin/index.html, viewed on February 29, 2000.

Joshi, R. R., and Selvam, A. M. (1999) Identification of self-organized criticality in atmospheric low frequency variability. *Fractals - Complex Geometry, Patterns, and Scaling in Nature and Society*, **4**, 421-425.

Koppány, G. Y. (1975) Estimation of the life span of atmospheric motion systems by means of atmospheric energetics. *Met. Mag*., **104**, 302-6.

Landsea, C. W. (2000) FAQ: Hurricanes, typhoons, and tropical cyclones (Version 2.8) http://www.aoml.noaa.gov/hrd/tcfaq/tcfaqHED.html, viewed on March 12, 2000.

Lovejoy, S. (1982) The area-perimeter relation for rain and cloud areas. *Science*, **216**, 185-187.

Lovejoy, S., and Mandelbrot, B. B. (1985) Fractal properties of rain, and a fractal model, *Tellus*, **37A**, 209-232.

Lovejoy, S., and Schertzer, D. (1986) Scale-invariance, symmetries, fractals, and stochastic simulations of atmospheric phenomena. *Bulletin of the American Meteorological Society*, **67**, 21-32.

Lovejoy, S. and Schertzer, D. (1990) Fractals, raindrops and resolution dependence of rain measurements. *Journal of Applied Meteorology*, **29**, 1167-1170.

Lovejoy, S., and Schertzer, D. (1997) Universal Multifractals Do Exist!: Comments on ‘‘A Statistical Analysis of Mesoscale Rainfall as a Random Cascade’’. *Journal of Applied Meteorology*, **36**, 1296-1303.

Malamud, B. D., Morein, G., and Turcotte, D. L. (1998) Forest Fires: An Example of Self-Organized Critical Behavior. *Science*, **281**, 1840-1842.

Rind, D. (1999) Complexity and climate. *Science*, **284**, 105-107.

Roberts, D. C., and Turcotte, D. L. (1999) Fractality and self-organized criticality of wars. *Fractals*, **6**, 351-357.

Scheinkman, J. A., and Woodford, M. (1994) Self-organized criticality and economic fluctuations. *American Journal of Economics*, **84**, 417.

Schroeder, M. (1991) *Fractals, chaos, power laws: minutes from an infinite paradise*. W.H. Freeman and Company, New York, 429 pp.

Selvam, A. M. (1990) Deterministic chaos, fractals and quantumlike mechanics in atmospheric flows, *Can. J. Phys.*, **68**, 831-841.

Selvam, A. M. (2000) Climate Dynamics, Chaos and Quantum Mechanics http://www.geocities.com/CapeCanaveral/Lab/5833/index.html, viewed on March 12, 2000.

Selvam, A. M., Pethkar, J. S., and Kulkarni, M. K. (1992) Signatures of a universal spectrum for interannual variability in rainfall time series over the Indian region. *Int. J. Climatol.*, **12**, 137-152.

Selvam, A. M., and Joshi, R. R. (1995) Universal spectrum for interannual variability in COADS global air and sea-surface temperatures, *Int. J. Climatol.*, **15**, 613-623.

Selvam, A. M., Pethkar, J. S., and Kulkarni, M. K., and Vijayakumar, R. (1996) Signatures of a universal spectrum for interannual variability in COADS surface pressure time series, *Int. J. Climatol.*, **16**, 393-404.

Storm Prediction Center – NOAA (2000) Climatological Data http://www.spc.noaa.gov/climo/, viewed on March 12, 2000.

Tessier, Y., Lovejoy, S., and Schertzer, D. (1993) Universal multifractals – theory and observations for rain and clouds. *Journal of Applied Meteorology*, **32**, 223-250.

The Tornado Project Online (2000) The Fujita Scale http://www.tornadoproject.com/, viewed on March 12, 2000.

Vattay, G., and Harnos, A. (1994) Scaling behavior in daily air humidity fluctuations. *Physical Review Letters*, **73**, 768-771.

Viegas, D. X. (1998) Weather, fuel status and fire occurrence: predicting large fires, pp. 31-48. In Moreno, J. M. (ed.) *Large Fires*, Backhuys Publishers, Leiden, The Netherlands.

Voss, R.V. and Clark, J. (1978) 1/f noise in music: Music from 1/f noise. *J. Acoust. Soc. Am.*, **63**, 258-263.

Walter, M. E. (1997) Simple mathematical patterns in complex situations (unpublished manuscript).

Figures

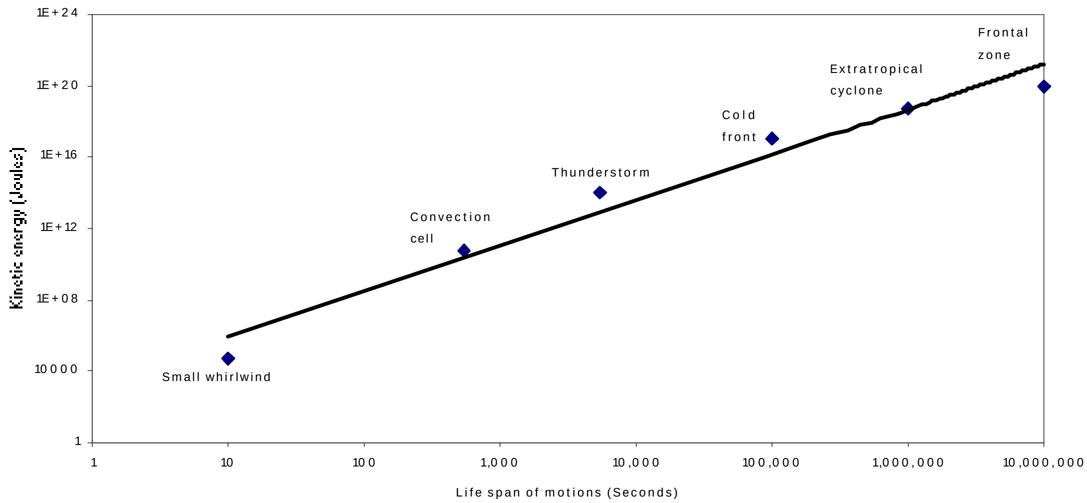

Figure 1. Life span of atmospheric motion systems as a function of kinetic energy (after Koppány 1975).

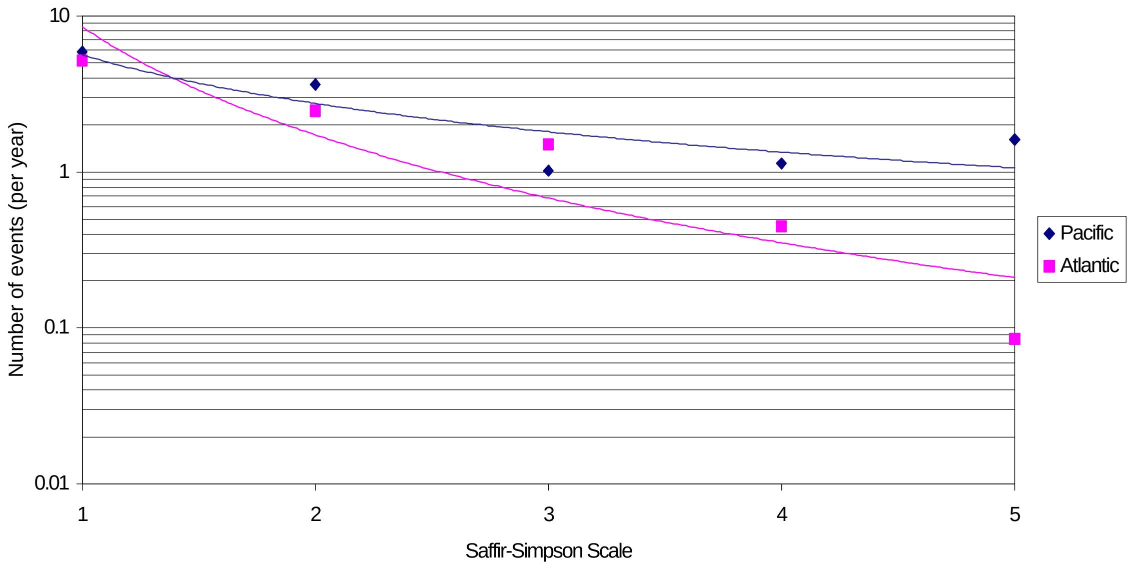

Figure 2. Distribution of historical US hurricanes from 1950-96. Data source: http://www.aoml.noaa.gov/hrd/basin/index.html

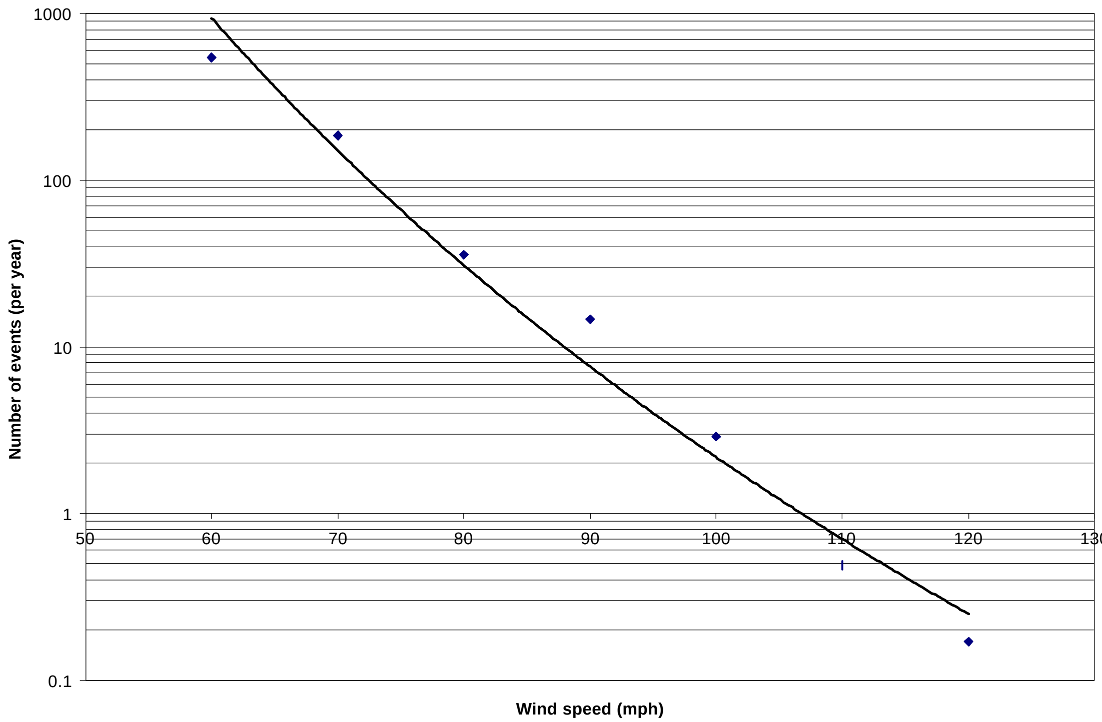

Figure 3. Distribution of historical US convective wind from 1955-95. Data source: http://www.spc.noaa.gov/archive/wind/

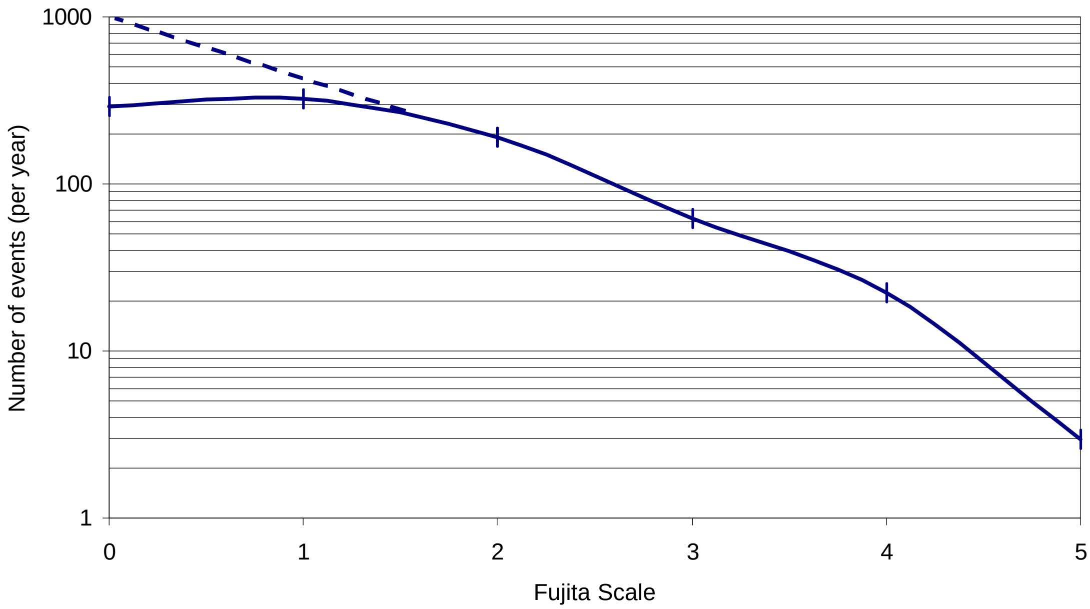

Figure 4. Distribution of historical US tornados from 1955-95. The full line represents actual data and the dashed portion represents what might be the "true" distribution of tornadoes. Data source: http://www.spc.noaa.gov/archive/tornadoes/

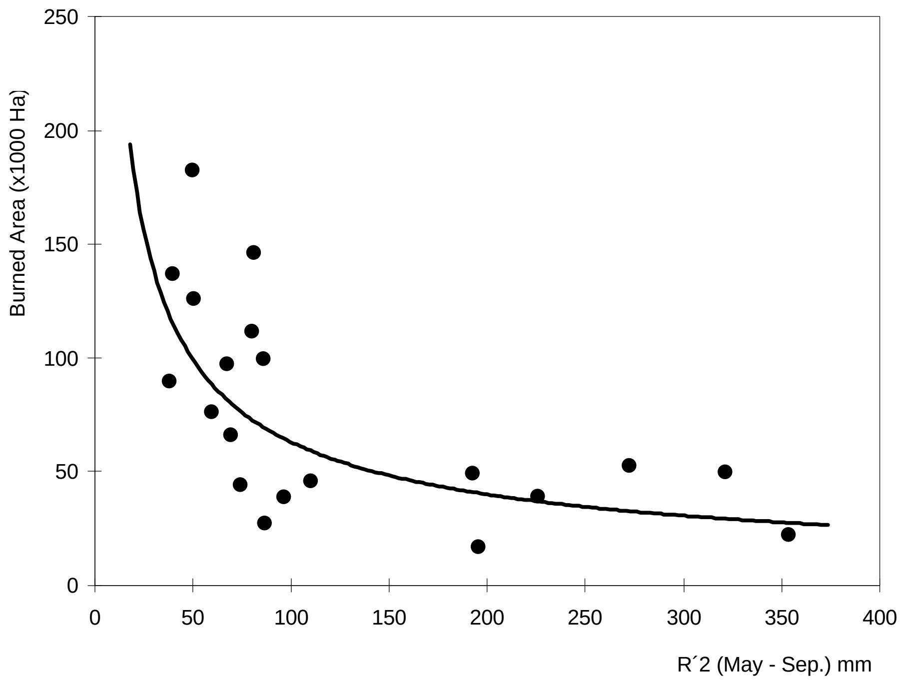

Figure 5. Burned surface in Portugal as a function of precipitation from June to September using data from 1975-1994 (Viegas, 1998).