

Data assimilation: a family of techniques broadly based on estimation theory

Most familiar in *forecast* applications

Also used in *analysis* applications

Most carbon cycle/biogeoscientific applications are analyses

Ecosystem assimilation strategy

$$NEE = f(\alpha_i, \beta_i, \rho_i)$$

α_i = forcing, examples are temperature and radiation.

ρ_i = time-invariant model parameters. Examples are the temperature exponent for respiration, light dependence of photosynthesis.

β_i are internal state variables of the system and are a function of prior integrated forcing (β_i can have long response times). Examples are foliar C and N, soil water, litter C and N.

Assimilation strategy

$$NEE = f(\alpha_i, \beta_i, \rho_i)$$

$$NEE = GPP(\alpha_{gpp}, \beta_{gpp}, \rho_{gpp}) - R(\alpha_r, \beta_r, \rho_r)$$

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$$R(\alpha_r, \beta_r, \rho_r) = R_a(\alpha_{ra}, \beta_{ra}, \rho_{ra}) + R_h(\alpha_{rh}, \beta_{rh}, \rho_{rh})$$

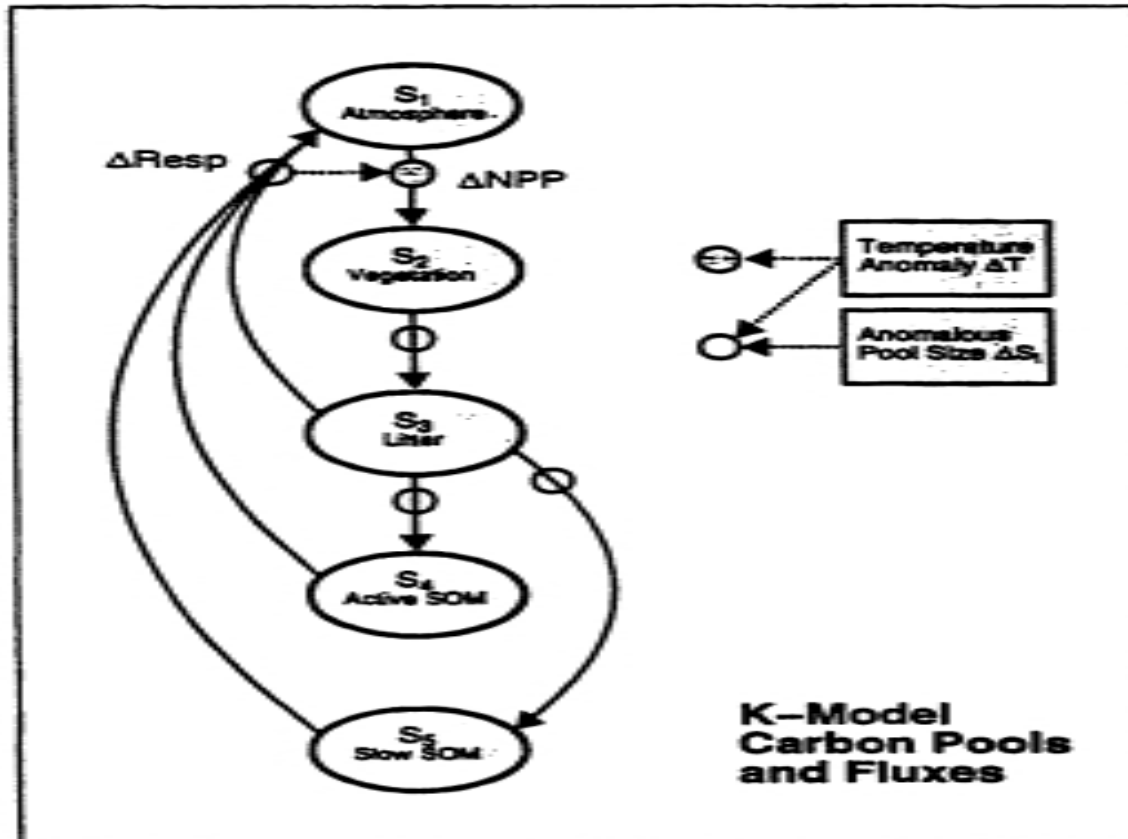
Two strategies based on estimation theory:

Parameter estimation:

$$\text{NEE} = f(\alpha_i, \beta_i, \rho_i)$$

State space estimation:

$$\text{NEE} = f(\alpha_i, \beta_i, \rho_i)$$



A simple model of the sensitivity of terrestrial ecosystem physiology to temperature

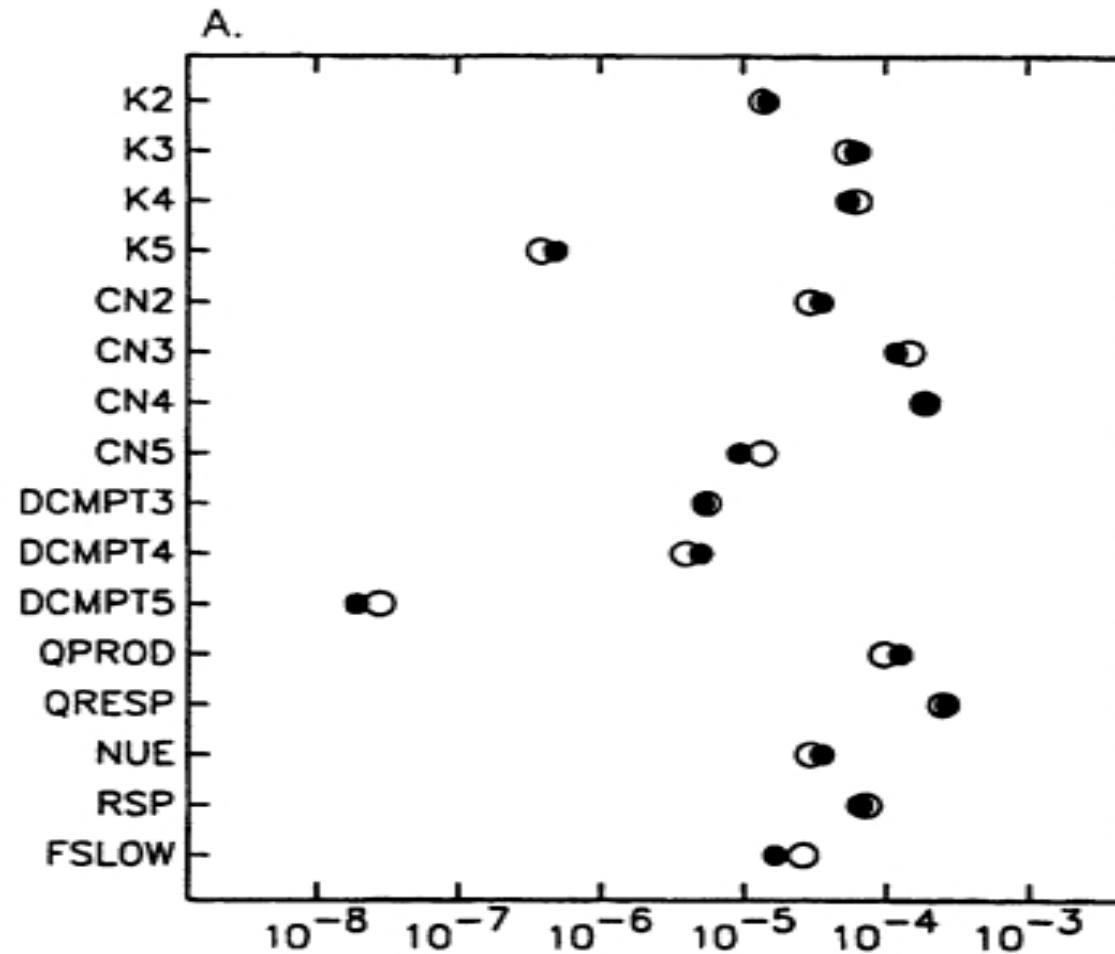
Parameters of the model and prior bounds

Table 1. *K-model parameters*

| Name | Description | Retrieved <i>N</i> -cycle | Retrieved no- <i>N</i> -cycle | Interval of acceptable values | Units |
|---------------|--------------------------------------|------------------------------|----------------------------------|----------------------------------|---|
| k_2 | vegetation turnover rate | 0.17 | 0.11 | 0.04–0.17 | month ⁻¹ |
| k_3 | litter turnover rate | 0.042 | 0.0083 | 0.0083–0.042 | month ⁻¹ |
| k_4 | active SOM turnover rate | 0.014 | 0.031 | 0.014–0.042 | month ⁻¹ |
| k_5 | passive SOM turnover rate | 0.00083 | 0.0042 | 0.00083–0.0042 | month ⁻¹ |
| cn_2 | C:N vegetation | 66.3 | N/A | 25.0–75.0 | gg ⁻¹ |
| cn_3 | C:n litter | 50.0 | N/A | 50.0–150.0 | gg ⁻¹ |
| cn_4 | C:N active SOM | 37.5 | N/A | 12.5–37.5 | gg ⁻¹ |
| cn_5 | C:N passive SOM | 22.5 | N/A | 7.5–22.5 | gg ⁻¹ |
| ϕ_3 | slope of k_3 versus temperature | 1.0 | 1.0 | 0.001–1.00 | month ⁻¹ deg ⁻¹ |
| ϕ_4 | slope of k_4 versus temperature | 1.0 | 1.0 | 0.001–1.00 | month ⁻¹ deg ⁻¹ |
| ϕ_5 | slope of k_5 versus temperature | 0.43 | 1.0 | 0.001–1.00 | month ⁻¹ deg ⁻¹ |
| λ_p | slope of NPP versus temperature | 1.66 | 1.22 | 0.001–2.00 | gm ⁻² year ⁻¹ deg ⁻¹ |
| λ_r | slope of resp. versus temperature | 2.00 | 1.60 | 0.001–2.00 | gm ⁻² year ⁻¹ deg ⁻¹ |
| \mathcal{F} | strength of <i>N</i> -feedback | 0.59 | N/A | 0.001–1.00 | dimensionless |
| \mathcal{D} | resp. fraction of decomposition | 0.45 | 0.49 | 0.20–0.80 | dimensionless |
| β | slow versus passive partitioning | 0.80 | 0.54 | 0.20–0.80 | dimensionless |

Retrieved parameter values using the K-model version with the *N*-cycle included are listed in the column labeled "retrieved *N*-cycle". The retrieved parameter values using the K-model version without the *N* cycle included are listed in the column labeled "retrieved no-*N*-cycle". The interval of acceptable values for parameters are listed in the column labeled "interval of acceptable values".

Model sensitivity from the adjoint: change in NEE for a 1% parameter change



Two strategies based on estimation theory:

Parameter estimation:

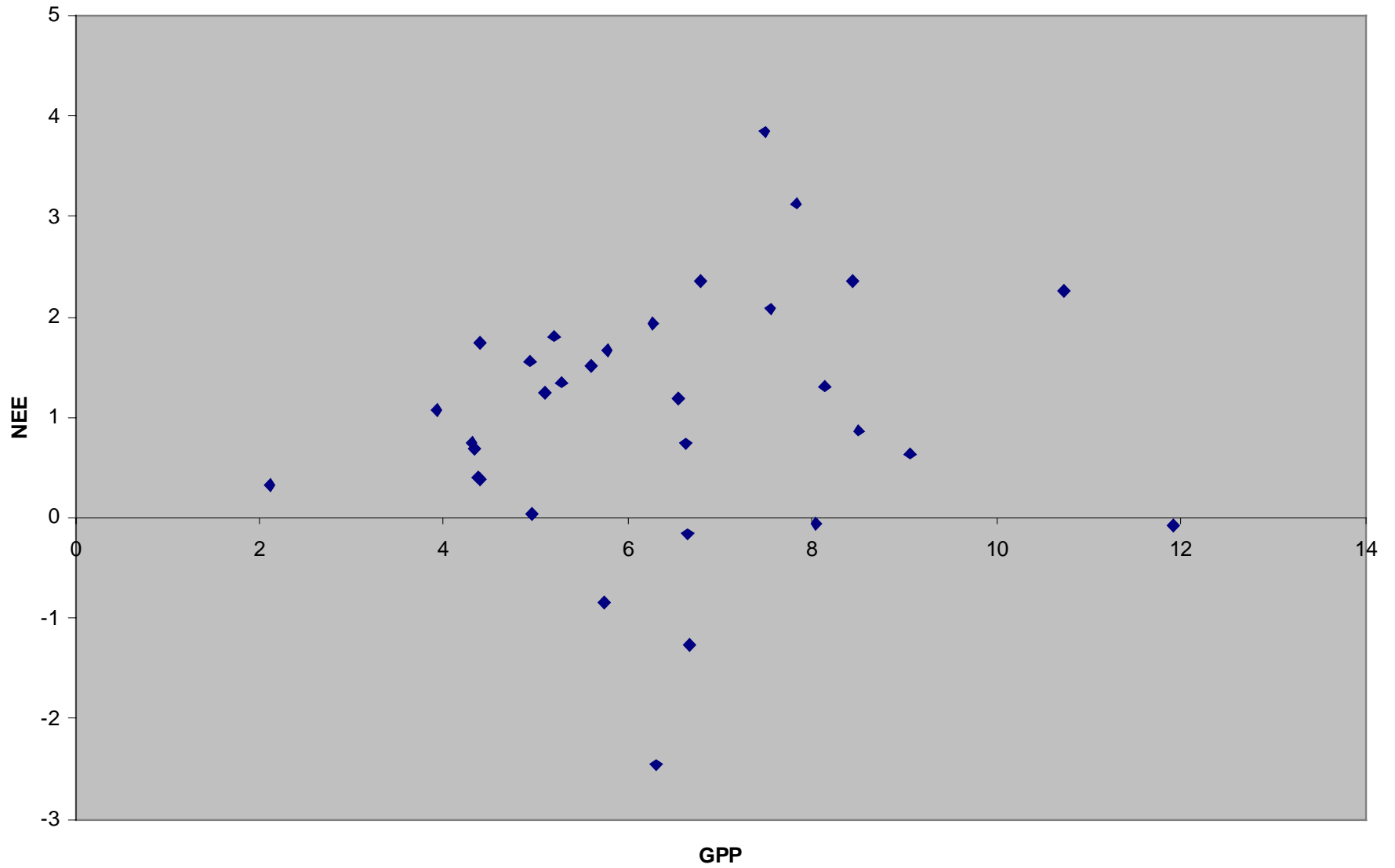
$$\text{NEE} = f(\alpha_i, \beta_i, \rho_i)$$

State space estimation:

$$\text{NEE} = f(\alpha_i, \beta_i, \rho_i)$$



GPP vs NEE



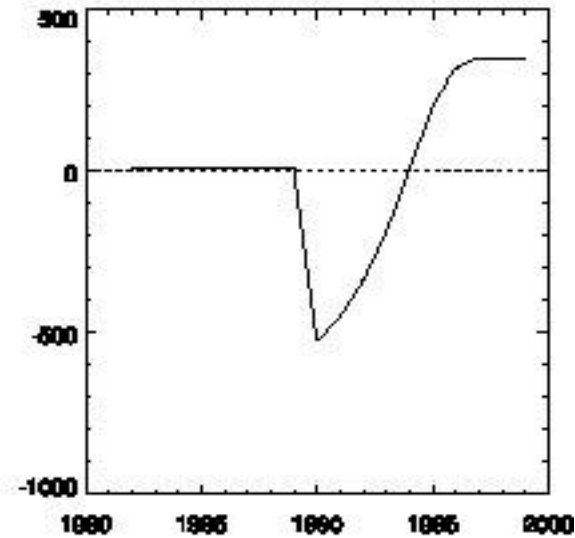
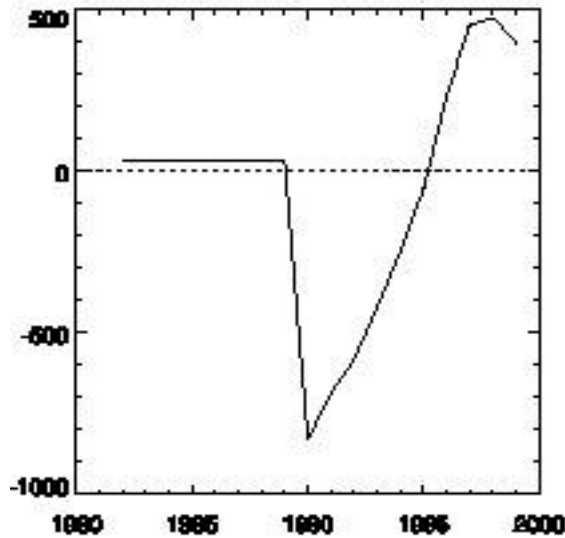
Average daily NEE and GPP are uncorrelated: On this time scale NEE reflects longer time scale processes and prior forcing

Blodgett Forest

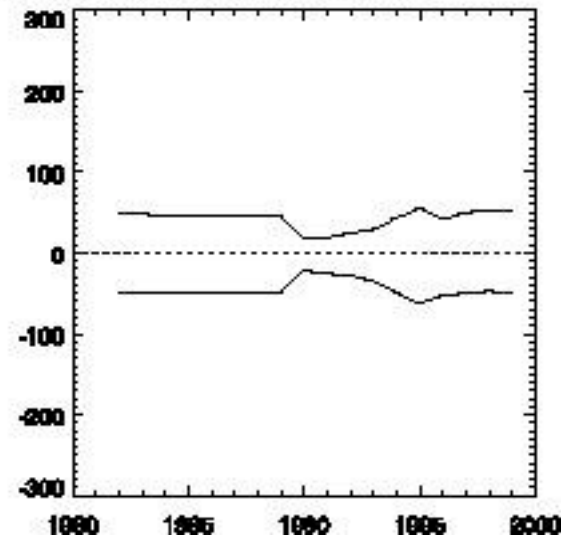
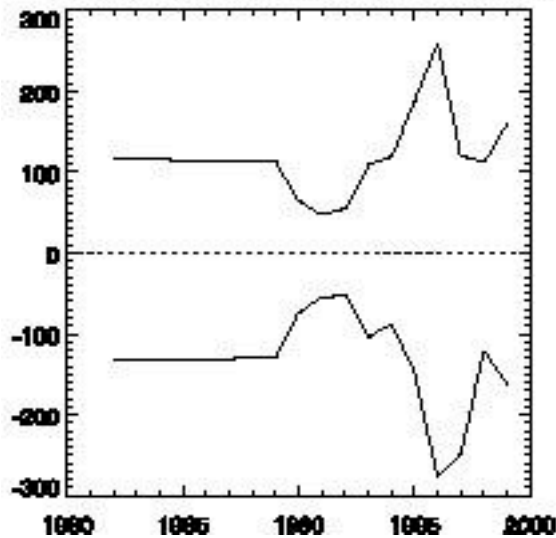
Florida slash pine

NEE depends
On ecosystem
state, on
multiple
time scales

Ensemble mean NEE (gC m² yr⁻¹)



Highest and lowest deviation from ensemble mean



Kalman Filter

(forward pass)

Observation equation:

$$NEE_{\text{obs}} = GPP - R_a - R_h$$

Gain Matrix

$\sigma(\text{model})$ from validation

$\sigma(\text{data})$ from u^*

Kalman Smoother

(backwards in time)

Uses

δNEE

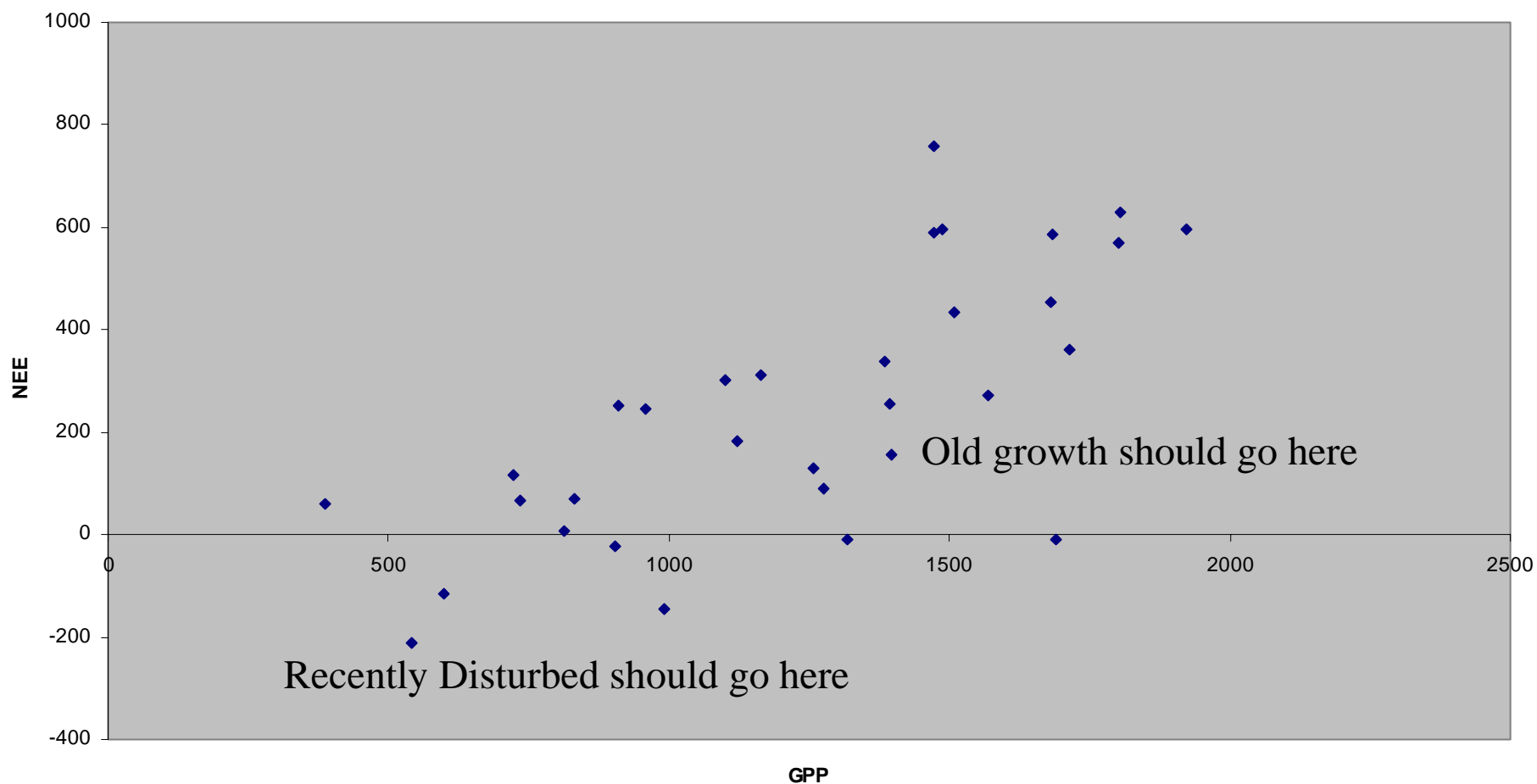
$\delta \beta_i$

State space estimation can help with both *model error* and *aggregation error*

The state space approach can be generalized to work with fluxes estimated from atmospheric and satellite measurements

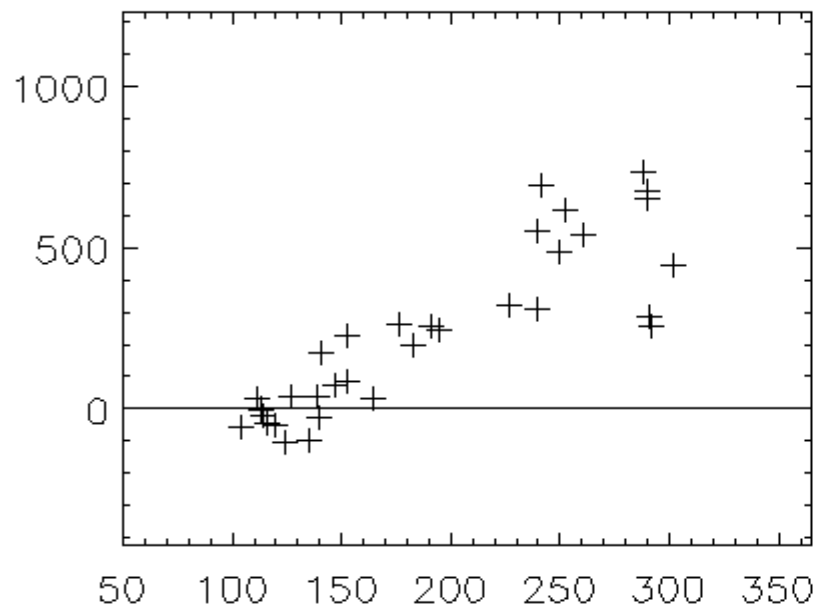
State space and parameter estimation techniques can be combined using ensemble data assimilation approaches

Annual NEE vs GPP

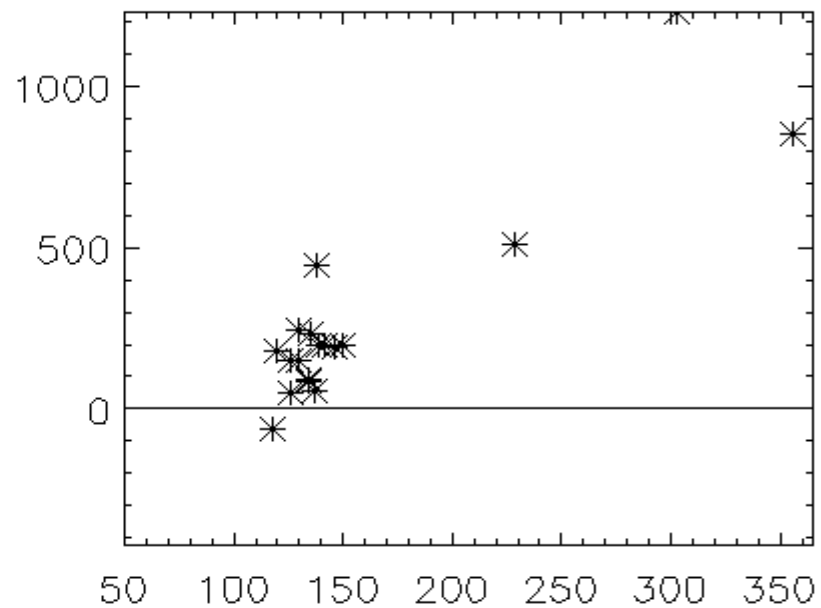


Annual NEE and GPP: also showing parts of “disturbance space”
unsampled in this data set

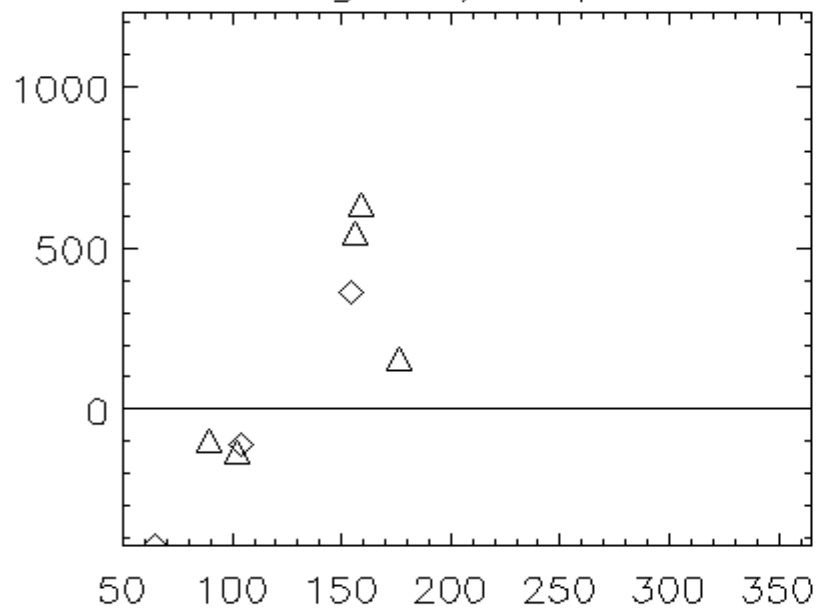
ENF



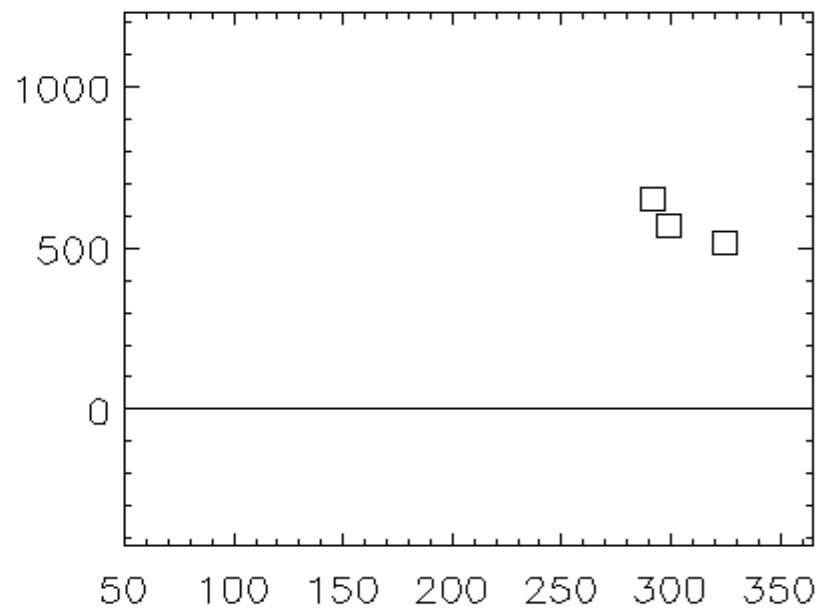
DBF



grass/crop

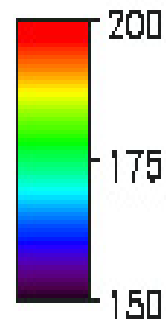
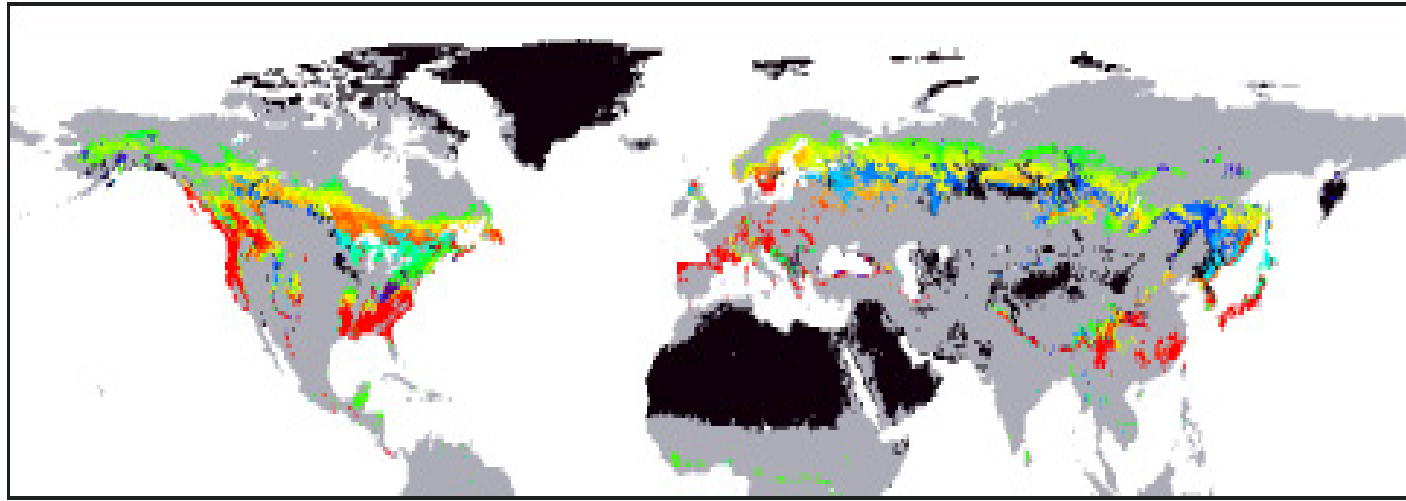


EBF



D.I.E.

Forest NEE extrapolated from CUP



Using: forest type map, separate regressions for broad and needle leafed forests and a satellite-based CUP, all aggregated to 0.5° .